SLIT–SLIDE–SEW BIJECTIONS

Jérémie BETTINELLI

March 19, 2019

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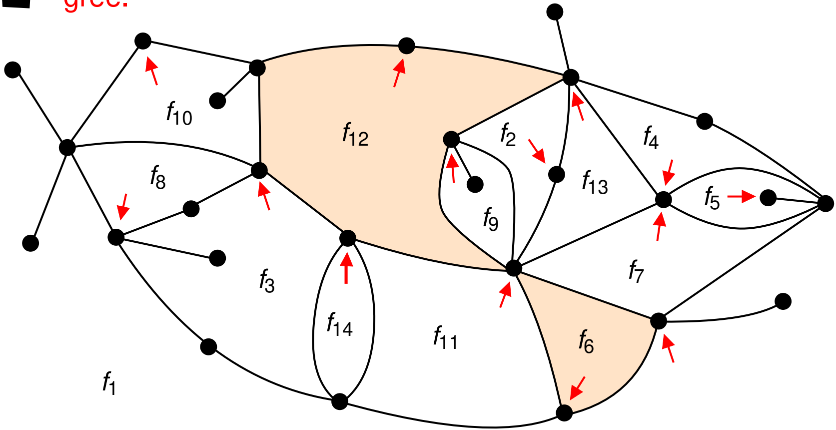

Tutte's formula of slicings

For $\boldsymbol{a} = (a_1, \dots, a_r) \in \mathbb{N}^r$, define the following.

- $M(\boldsymbol{a})$: number of plane maps with r numbered faces $f_1, \dots, f_r$ of respective degrees $a_1, \dots, a_r$, with a marked corner per face.
- $E(\boldsymbol{a}) := \frac{1}{2}\sum_{i=1}^{r} a_i$: numbers of edges of maps of type $\boldsymbol{a}$.
- $V(\boldsymbol{a}) := E(\boldsymbol{a}) - r + 2$: numbers of vertices of maps of type $\boldsymbol{a}$.

Theorem (Formula of slicings, [Tutte '62])

For bipartite or quasibipartite maps (i.e., at most two odd a_i's),

$$M(\boldsymbol{a}) = \frac{(E(\boldsymbol{a})-1)!}{V(\boldsymbol{a})\,!}\prod_{i=1}^{r}\alpha(a_i)\,, \quad \text{where} \quad \alpha(x) := \frac{x!}{\lfloor x/2\rfloor!\,\lfloor (x-1)/2\rfloor!}\,.$$

transfer bijections [Cori '75], encoding by blossoming trees [Schaeffer '97], Bouttier–Di Francesco–Guitter bijection [Collet–Fusy '14]

How to grow a bipartite map

Proposition (Adding two corners to the same face)

Let $\boldsymbol{a} = (a_1, \ldots, a_r)$ be an r-tuple of positive even integers and let $\tilde{\boldsymbol{a}} = (\tilde{a}_1, \ldots, \tilde{a}_r) := (a_1 + 2, a_2, \ldots, a_r)$. Then,

$$(a_1 + 1)\,(a_1 + 2)\,E(\boldsymbol{a})\,M(\boldsymbol{a}) = \left\lfloor \tilde{a}_1/2 \right\rfloor \left\lfloor (\tilde{a}_1 - 1)/2 \right\rfloor\,V(\tilde{\boldsymbol{a}})\,M(\tilde{\boldsymbol{a}}).$$

Proposition (Adding one corner to each of two different faces)

Let $\boldsymbol{a} = (a_1, \ldots, a_r)$ be an r-tuple of positive even integers and let $\tilde{\boldsymbol{a}} = (\tilde{a}_1, \ldots, \tilde{a}_r) := (a_1 + 1, a_2 + 1, a_3, \ldots, a_r)$. Then,

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except for $a_i = 1$, recover Tutte's formula of slicings by subsequent applications from the initial condition $M(2, \ldots, 2) = 2^{r-1}(r-1)!$

Related works

- [Cori '75, '76]. Counting by transferring one degree from a face to a neighboring one.
- [Louf '18]. Bijections interpreting KP hierarchy formulas, which strongly rely on the mechanism of sliding along a local path. Also possible disconnection of the map resulting in two output maps, which corresponds to quadratic formulas.

Today's specials

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MENU REMINDER AND GENERALIZATION OF THE PREVIOUS COMPLICATED BIJECTIONS

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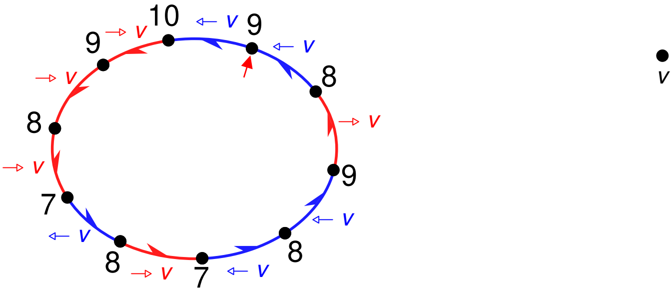
MENU ANSWER TO EACH AND EVERY QUESTION ANDREA EVER ASKED DURING ALEA FROM 2010 TO 2015, A MENU THAT ONLY ANDREA CAN UNDERSTAND

Today's specials

Preliminary facts

Proposition

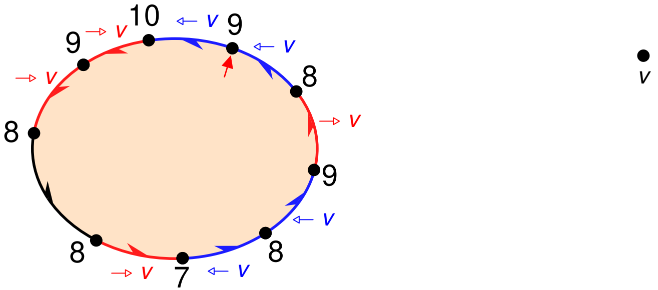
In a bipartite map, no edge can be parallel to a vertex. More precisely, for any given face and any given vertex, exactly half of the half-edges incident to the face are directed toward the vertex, the other half being directed away from the vertex.



Preliminary facts

Proposition

In a quasibipartite map, a cycle has odd length if and only if it separates the two odd-degree faces. Moreover, for any given vertex v, among the a half-edges incident to an odd-degree face, exactly one is parallel to v, $(a-1)/2$ are directed toward v and $(a-1)/2$ are directed away from v.



Adding two corners to the same face in a bipartite map

$\boldsymbol{a} = (a_1, \ldots, a_r) \in 2\mathbb{N}^r$ $\tilde{\boldsymbol{a}} = (\tilde{a}_1, \ldots, \tilde{a}_r) := (a_1 + 2, a_2, \ldots, a_r)$

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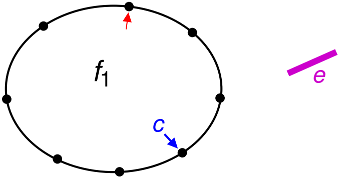

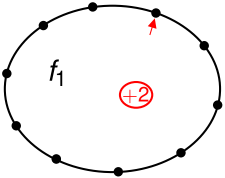

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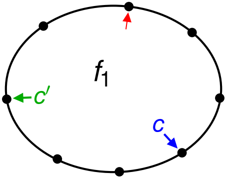

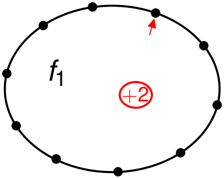

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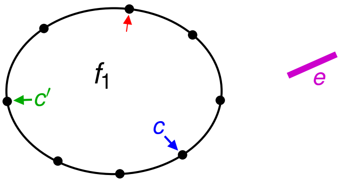

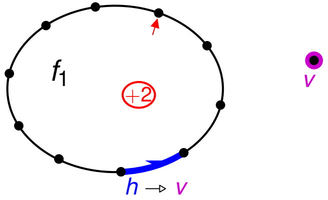

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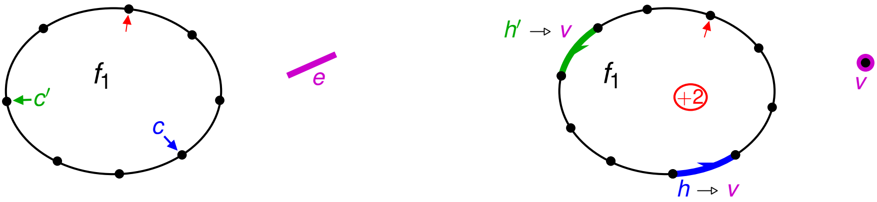

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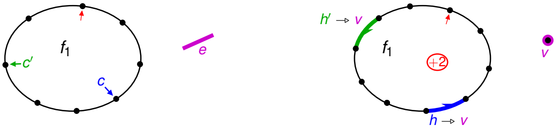

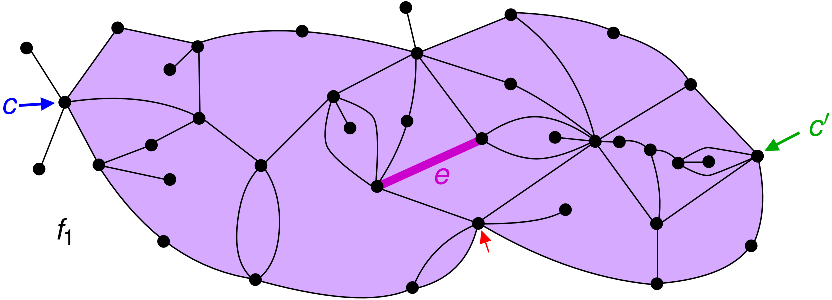

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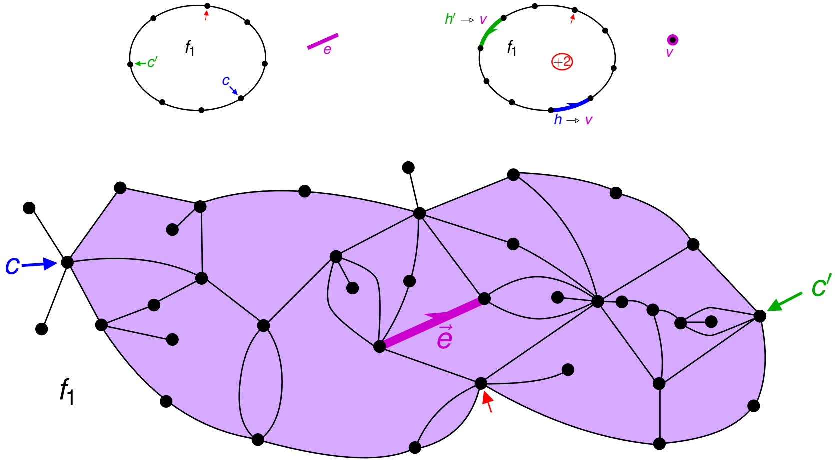
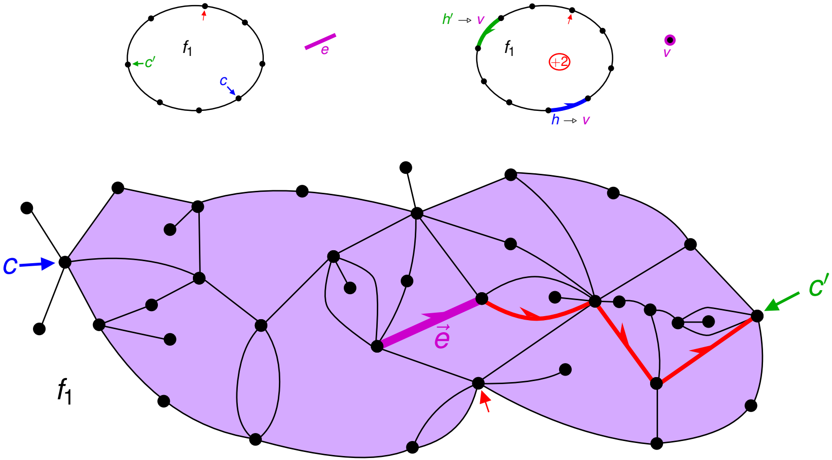


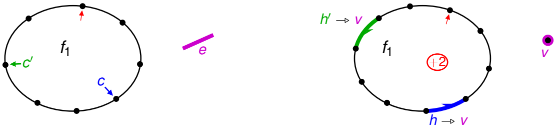
Orient e toward c'.

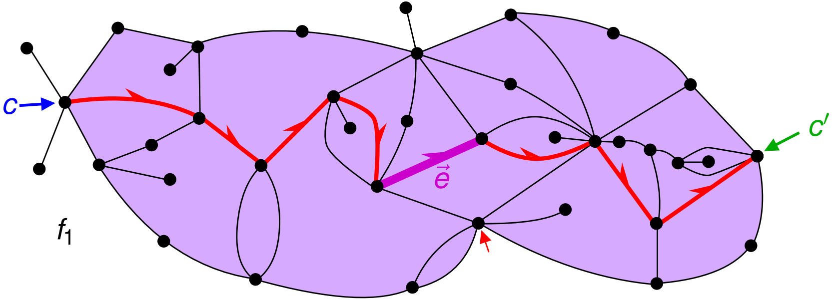
Adding two corners to the same face in a bipartite map



Consider the rightmost geodesic from $\vec{e}$ toward c'.

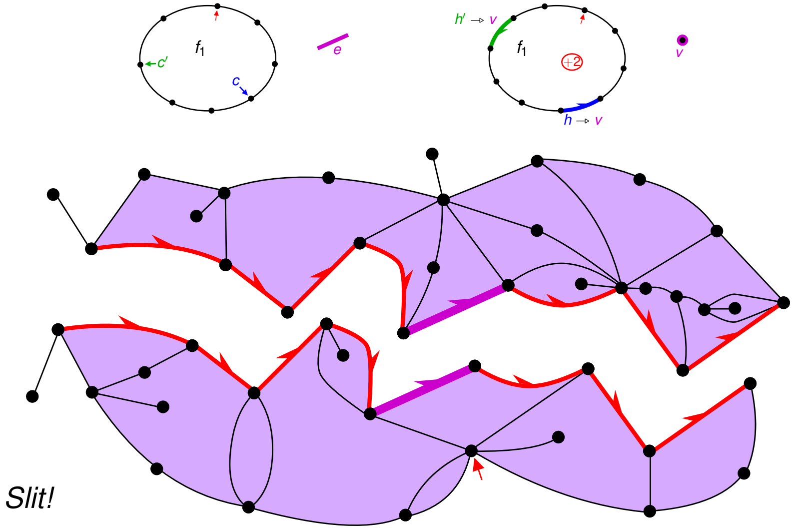
Adding two corners to the same face in a bipartite map





Do "the same" with c instead of c'.

Adding two corners to the same face in a bipartite map



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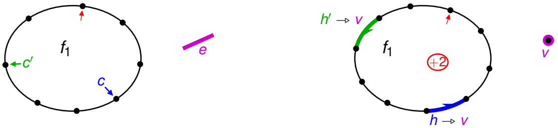
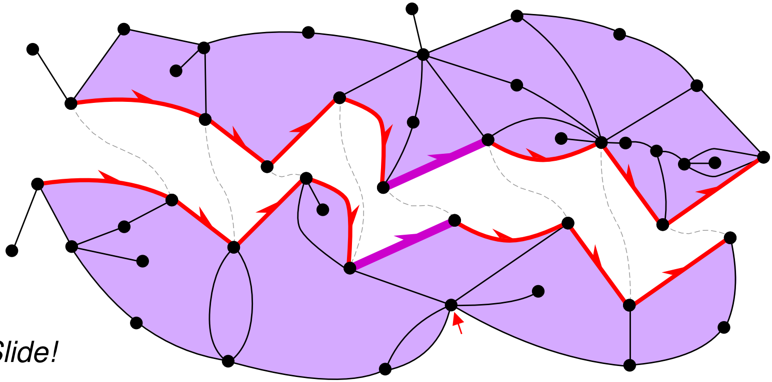
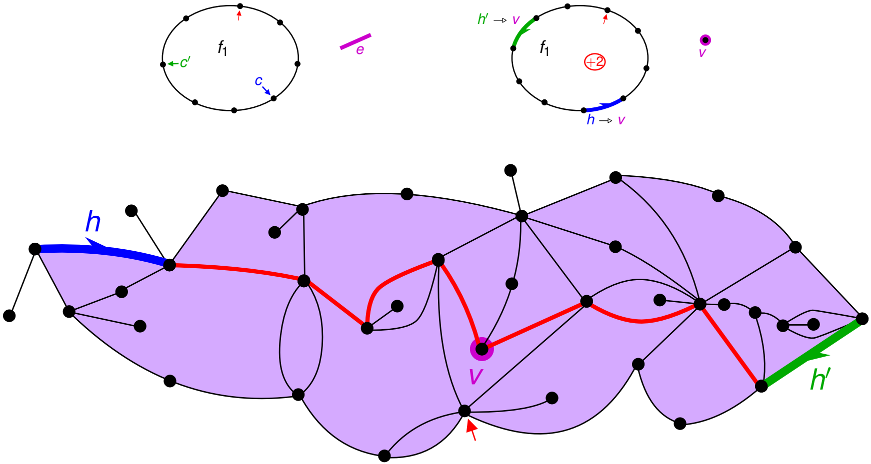




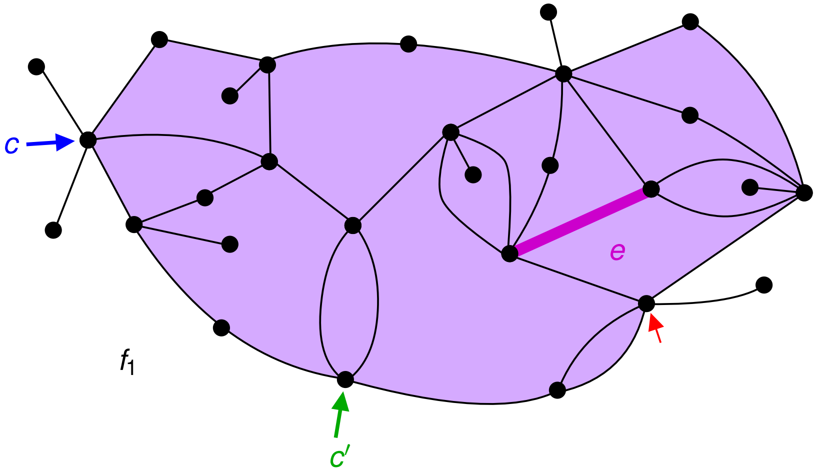
Slide!

Adding two corners to the same face in a bipartite map



Sew! And mark v, h and h'.

Pinched case



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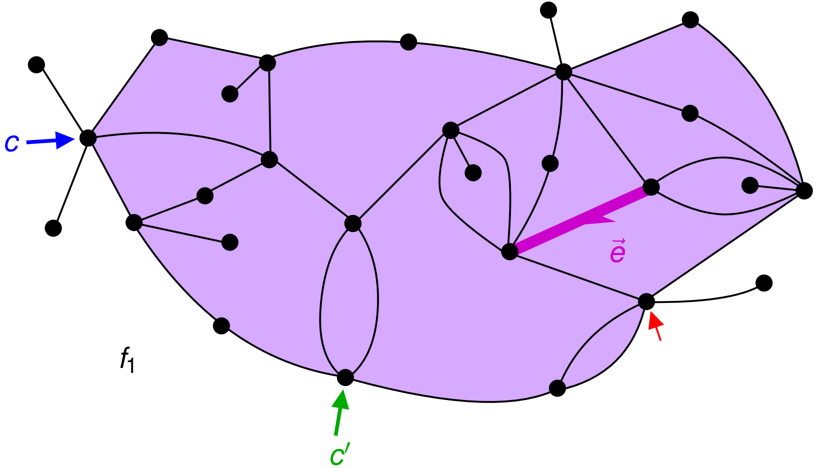

Orient e *toward* c'.

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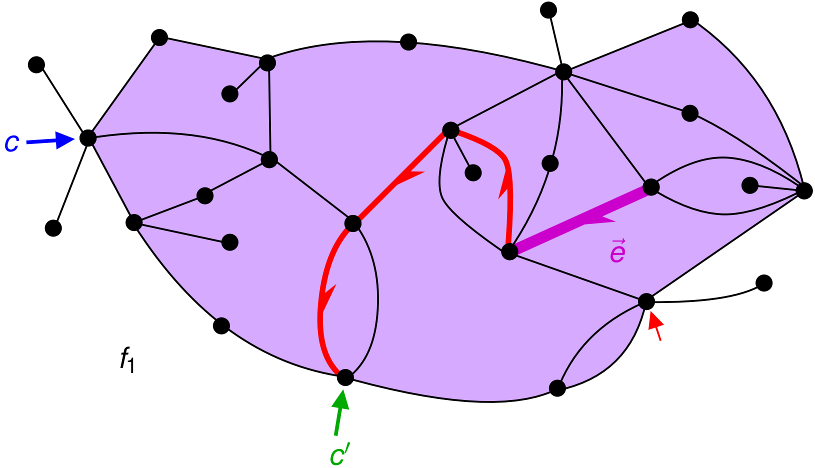

Consider the rightmost geodesic from $\vec{e}$ toward c'.

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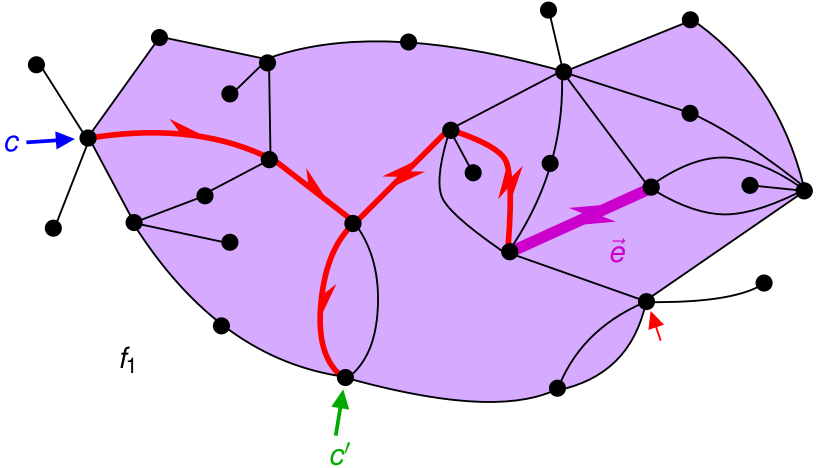

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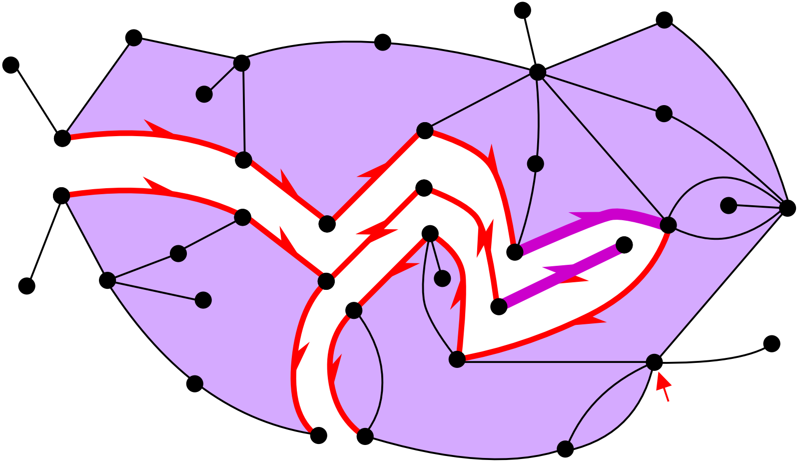
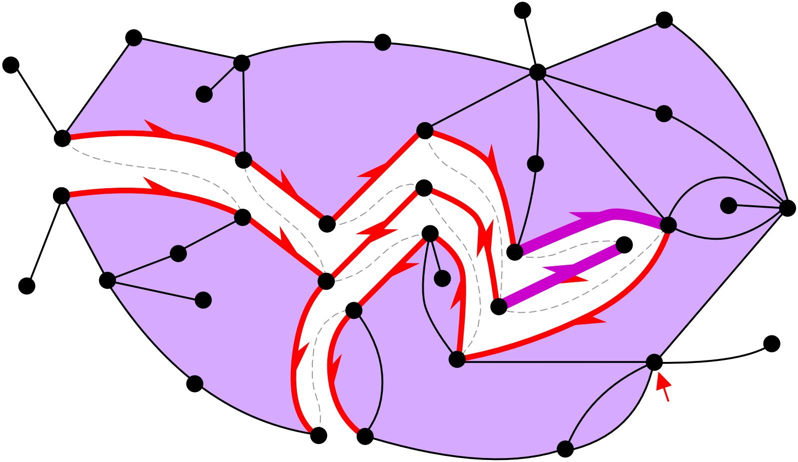


Slit!

Pinched case



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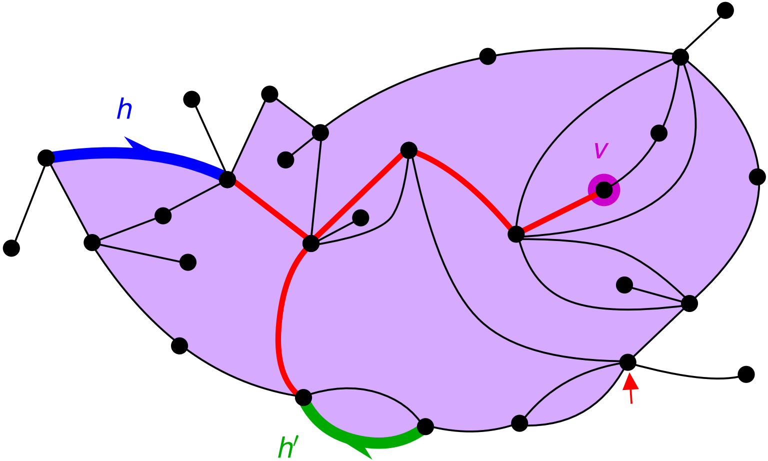

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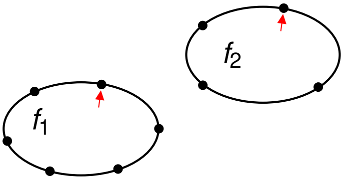

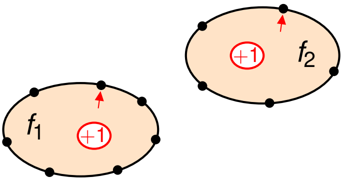

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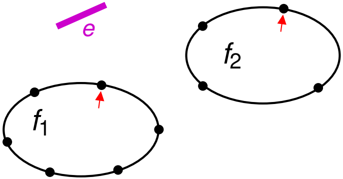

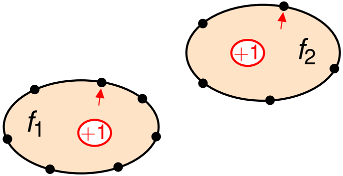

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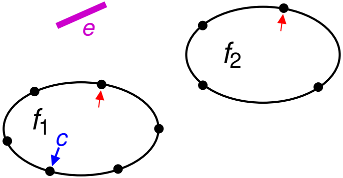

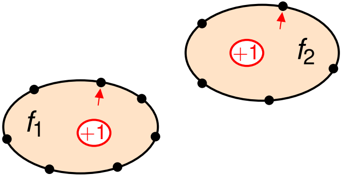

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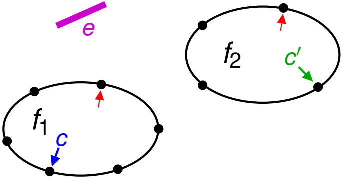

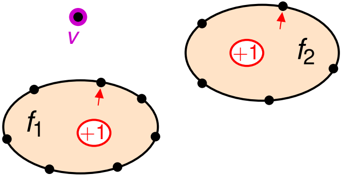

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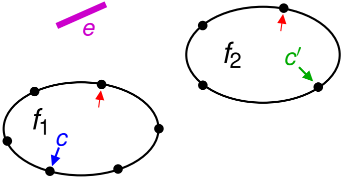
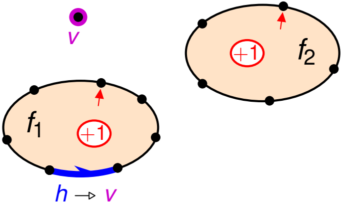
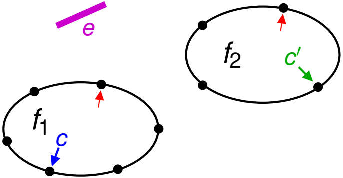


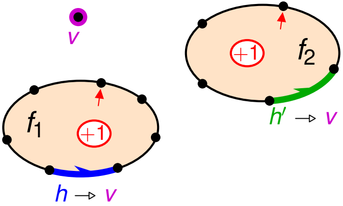


Adding one corner to two faces in a bipartite map

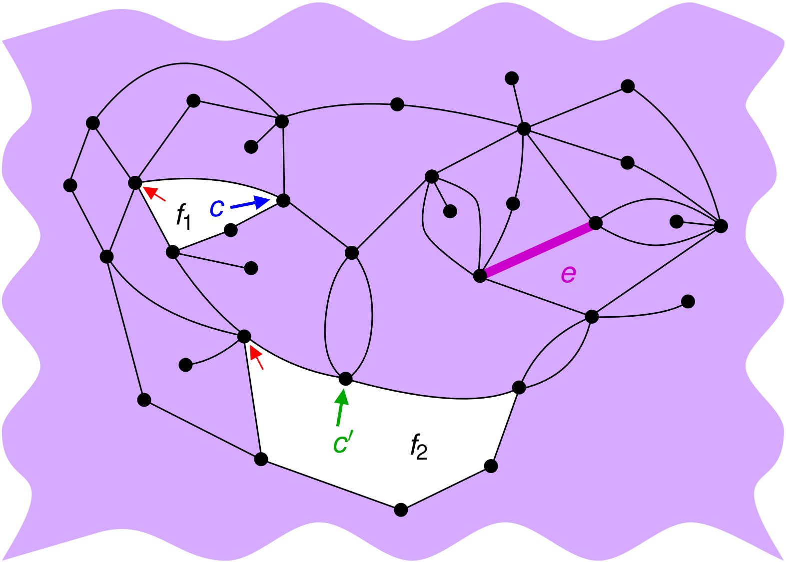
$\boldsymbol{a} = (a_1, \dots, a_r) \in 2\mathbb{N}^r$ $\qquad$ $\tilde{\boldsymbol{a}} := (a_1 + 1, a_2 + 1, a_3, \dots, a_r)$

$$\underbrace{(a_1+1)}_{\substack{\text{corner}\\ c \text{ in } f_1}}\underbrace{(a_2+1)}_{\substack{\text{corner}\\ c' \text{ in } f_2}}\underbrace{E(\boldsymbol{a})}_{\text{edge}}\,M(\boldsymbol{a}) = \underbrace{\lfloor \tilde{a}_1/2 \rfloor}_{\substack{\text{half-edge } h\\ \text{of } f_1 \text{ toward } v}}\;\underbrace{\lfloor \tilde{a}_2/2 \rfloor}_{\substack{\text{half-edge } h'\\ \text{of } f_2 \text{ toward } v}}\;\underbrace{V(\tilde{\boldsymbol{a}})}_{\text{vertex } v}\,M(\tilde{\boldsymbol{a}})$$

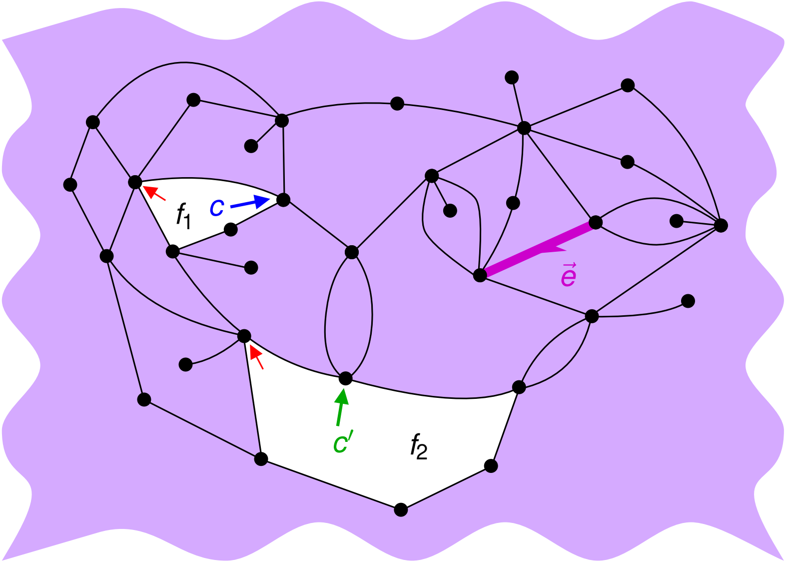




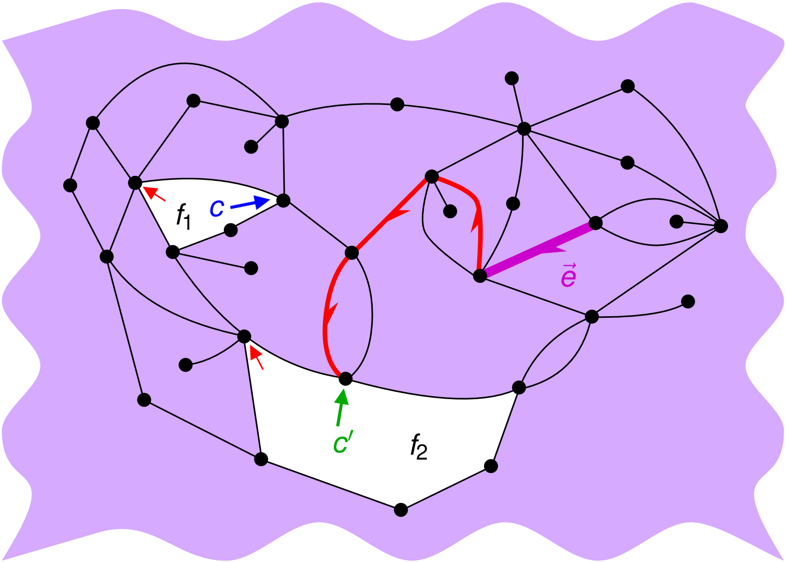
Adding one corner to two faces in a bipartite map



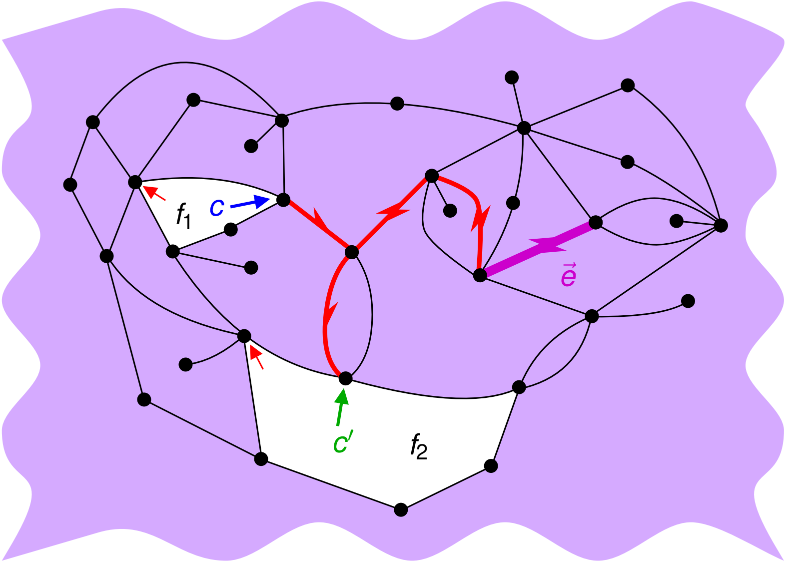
Adding one corner to two faces in a bipartite map



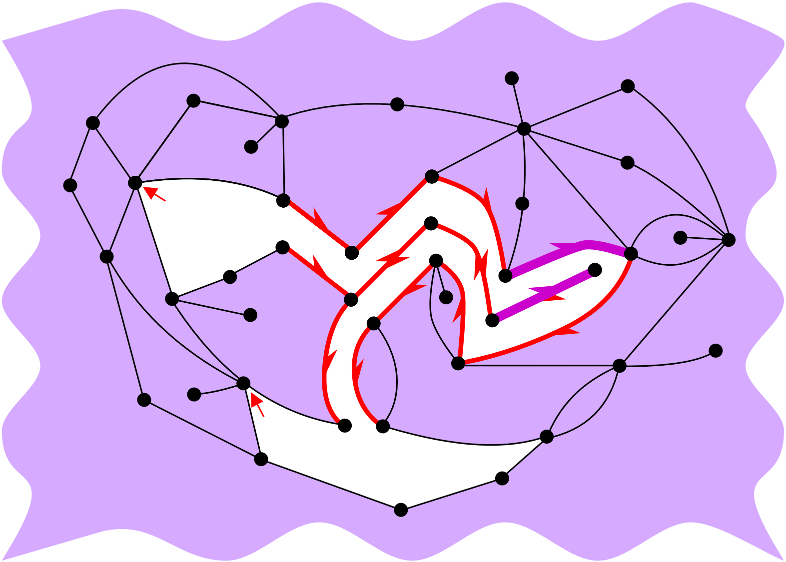
Adding one corner to two faces in a bipartite map



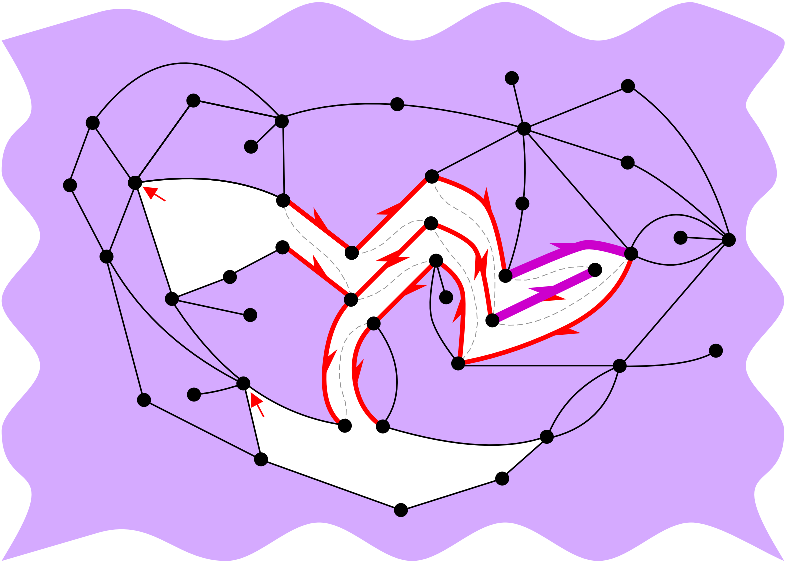
Adding one corner to two faces in a bipartite map



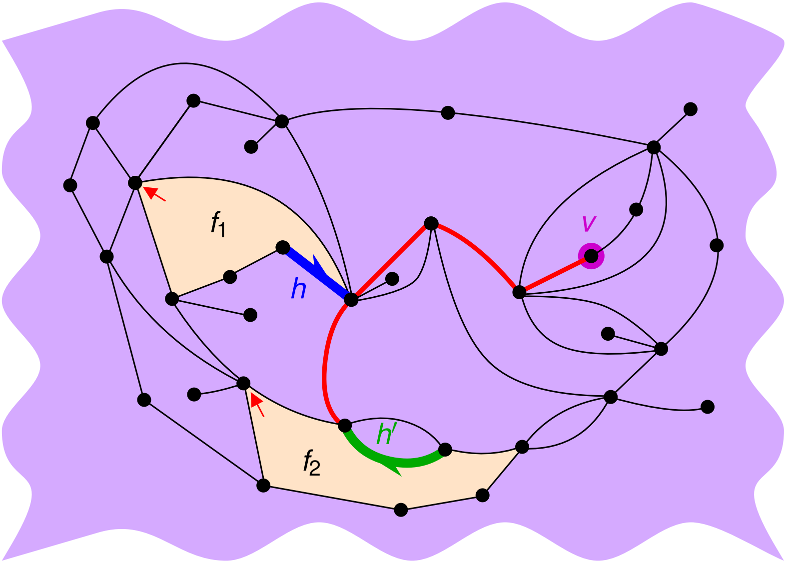
Adding one corner to two faces in a bipartite map



Adding one corner to two faces in a bipartite map



Adding one corner to two faces in a bipartite map



Today's specials

MENU

MENU IN FACT, IT IS NOT THAT COMPLICATED

MENU ANSWER TO EACH AND EVERY QUESTION ANDREA EVER ASKED DURING ALEA FROM 2010 TO 2015, A MENU THAT ONLY ANDREA CAN UNDERSTAND

Today's specials

MENU

MENU In fact, it is not that complicated

MENU Answer to each and every question Andrea ever asked during ALEA from 2010 to 2015, a menu that only Andrea can understand

How to transfer a corner

Proposition (from a face of degree at least 2)

Let $\boldsymbol{a} = (a_1, \dots, a_{r+1}) \in \mathbb{N}^{r+1}$ *with* $a_{r+1} \geq 2$ *such that*

- *either every* a_i *is even;*
- *or only* a_{r+1} *and one other* a_i *are odd.*

Let $\tilde{\boldsymbol{a}} = (\tilde{a}_1, \dots, \tilde{a}_{r+1}) := (a_1 + 1, a_2, \dots, a_r, a_{r+1} - 1)$. *Then,*

$$(a_1 + 1) \lfloor a_{r+1}/2 \rfloor M(\boldsymbol{a}) = \lfloor \tilde{a}_1/2 \rfloor (\tilde{a}_{r+1} + 1) M(\tilde{\boldsymbol{a}}).$$

Proposition (from a degree 1-face)

Let $\boldsymbol{a} = (a_1, \dots, a_r, 1)$ *be an* $r+1$*-tuple of positive integers with two odd coordinates and let* $\tilde{\boldsymbol{a}} = (\tilde{a}_1, \dots, \tilde{a}_r) := (a_1 + 1, a_2, \dots, a_r)$. *Then,*

$$(a_1 + 1) M(\boldsymbol{a}) = \lfloor \tilde{a}_1/2 \rfloor V(\tilde{\boldsymbol{a}}) M(\tilde{\boldsymbol{a}}).$$

Transferring from a face of degree at least 2

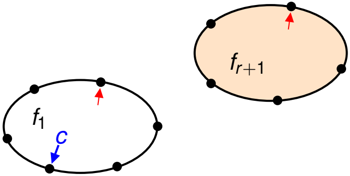
$$\boldsymbol{a} = (a_1, \dots, a_{r+1}) \qquad \tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \dots, a_r, a_{r+1} - 1)$$

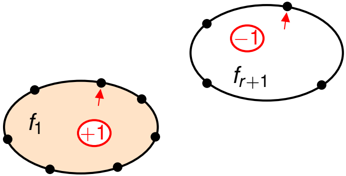
$$(a_1 + 1) \left\lfloor a_{r+1}/2 \right\rfloor M(\boldsymbol{a}) = \left\lfloor \tilde{a}_1/2 \right\rfloor (\tilde{a}_{r+1} + 1)\, M(\tilde{\boldsymbol{a}})$$

Transferring from a face of degree at least 2

$\boldsymbol{a} = (a_1, \dots, a_{r+1})$ $\qquad$ $\tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \dots, a_r, a_{r+1} - 1)$

$$\underbrace{(a_1 + 1)}_{\substack{\text{corner} \\ c \text{ in } f_1}} \left\lfloor a_{r+1}/2 \right\rfloor M(\boldsymbol{a}) = \left\lfloor \tilde{a}_1/2 \right\rfloor (\tilde{a}_{r+1} + 1)\, M(\tilde{\boldsymbol{a}})$$





Transferring from a face of degree at least 2

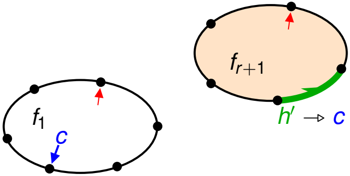
$\boldsymbol{a} = (a_1, \dots, a_{r+1})$ $\tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \dots, a_r, a_{r+1} - 1)$

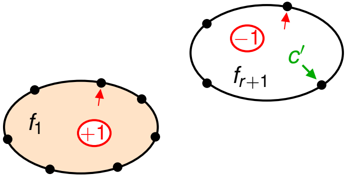
$$\underbrace{(a_1 + 1)}_{\substack{\text{corner} \\ c \text{ in } f_1}} \left\lfloor a_{r+1}/2 \right\rfloor M(\boldsymbol{a}) = \left\lfloor \tilde{a}_1/2 \right\rfloor \underbrace{(\tilde{a}_{r+1} + 1)}_{\substack{\text{corner} \\ c' \text{ in } f_{r+1}}} M(\tilde{\boldsymbol{a}})$$

Transferring from a face of degree at least 2

$$\boldsymbol{a} = (a_1, \dots, a_{r+1}) \qquad \tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \dots, a_r, a_{r+1} - 1)$$

$$\underbrace{(a_1 + 1)}_{\substack{\text{corner} \\ c \text{ in } f_1}} \underbrace{\lfloor a_{r+1}/2 \rfloor}_{\substack{\text{half-edge } h' \\ \text{of } f_{r+1} \text{ toward } c}} M(\boldsymbol{a}) = \lfloor \tilde{a}_1/2 \rfloor \underbrace{(\tilde{a}_{r+1} + 1)}_{\substack{\text{corner} \\ c' \text{ in } f_{r+1}}} M(\tilde{\boldsymbol{a}})$$

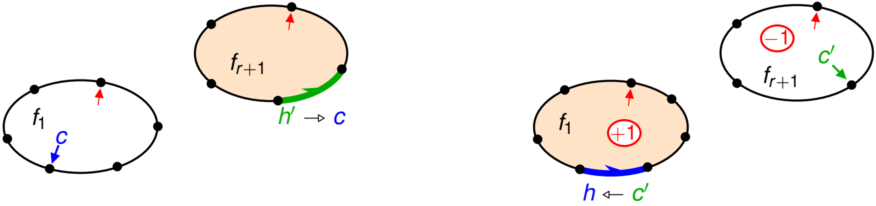




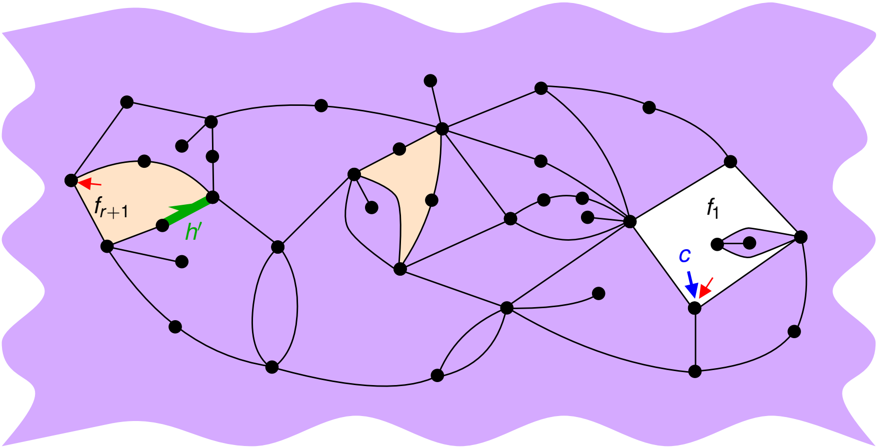
Transferring from a face of degree at least 2

$$\boldsymbol{a} = (a_1, \dots, a_{r+1}) \qquad \tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \dots, a_r, a_{r+1} - 1)$$

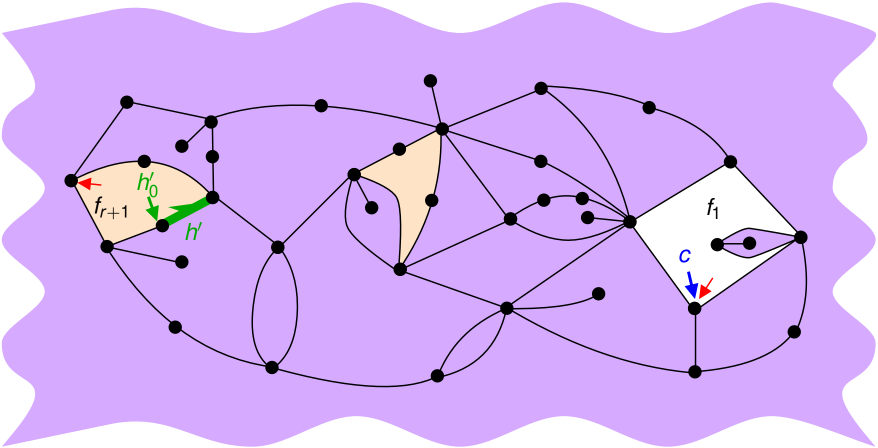
$$\underbrace{(a_1 + 1)}_{\substack{\text{corner} \\ c \text{ in } f_1}} \underbrace{\lfloor a_{r+1}/2 \rfloor}_{\substack{\text{half-edge } h' \\ \text{of } f_{r+1} \text{ toward } c}} M(\boldsymbol{a}) = \underbrace{\lfloor \tilde{a}_1/2 \rfloor}_{\substack{\text{half-edge } h \\ \text{of } f_1 \text{ away from } c'}} \underbrace{(\tilde{a}_{r+1} + 1)}_{\substack{\text{corner} \\ c' \text{ in } f_{r+1}}} M(\tilde{\boldsymbol{a}})$$



Transferring from a face of degree at least 2

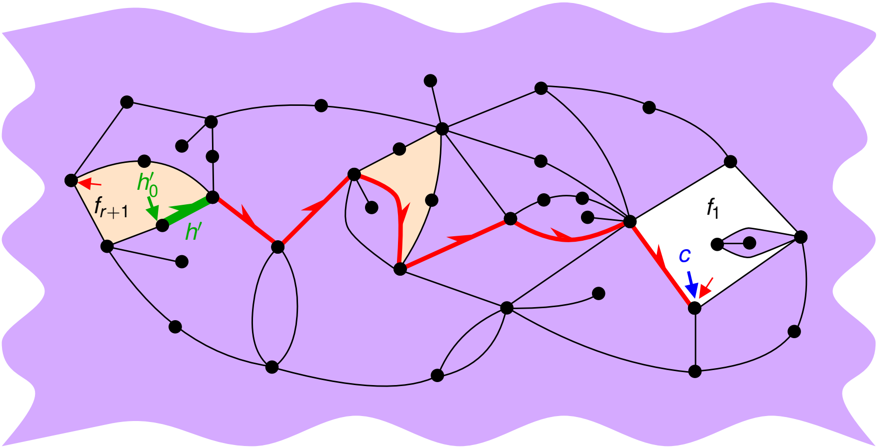


Transferring from a face of degree at least 2



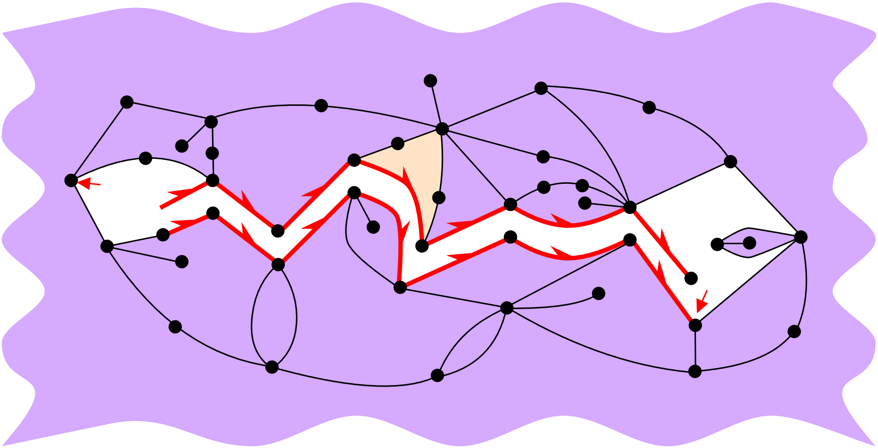
Consider the corner h'_0 delimited by h' and its predecessor in f_{r+1}.

Transferring from a face of degree at least 2



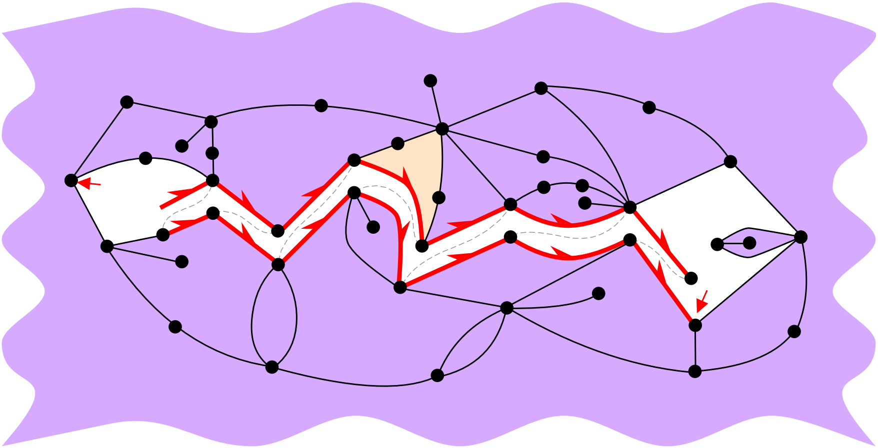
Consider the leftmost geodesic from h'_0 toward c.

Transferring from a face of degree at least 2



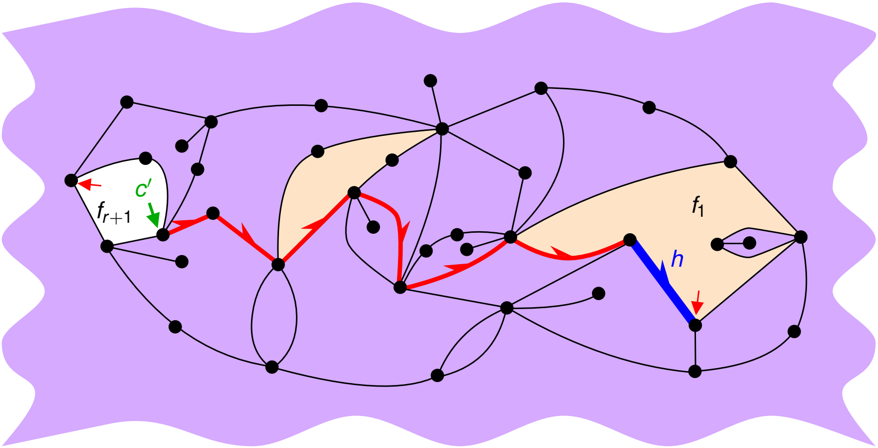
Slit!

Transferring from a face of degree at least 2



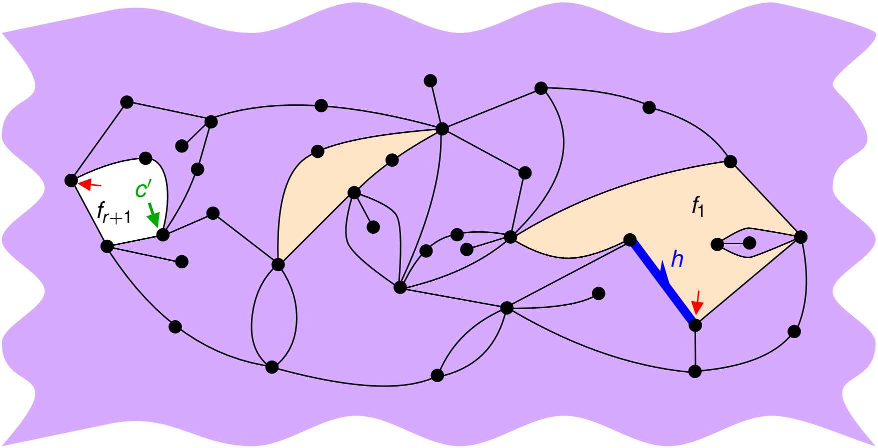
Slide!

Transferring from a face of degree at least 2

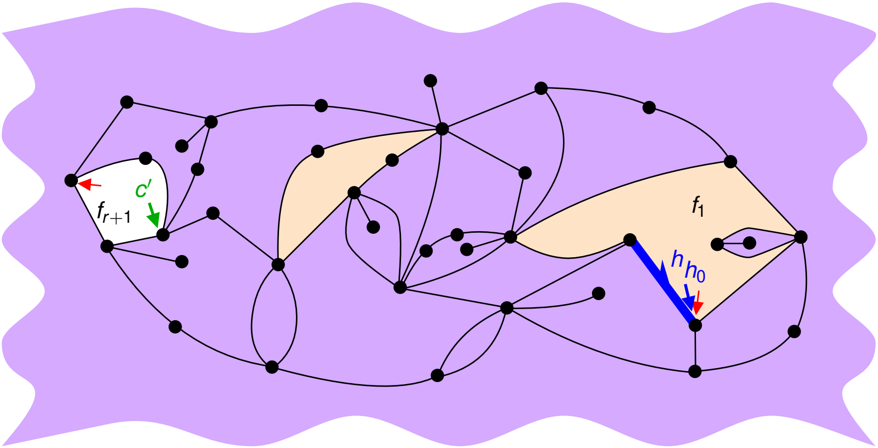


Sew! And mark h and c'.

Conversely

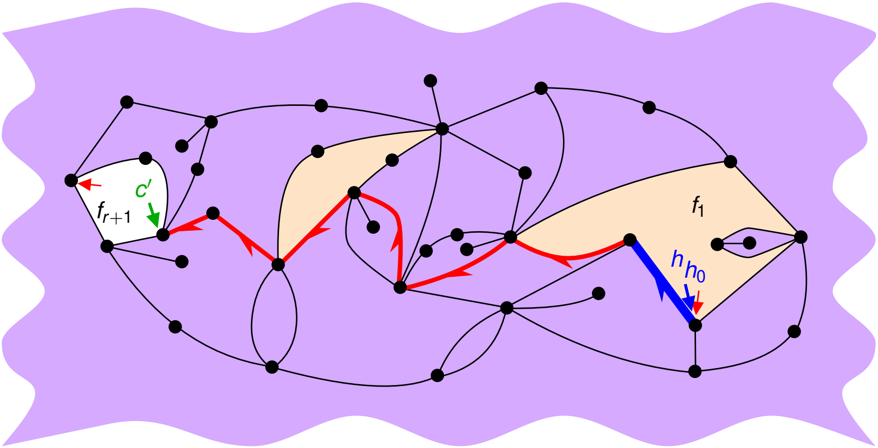


Conversely



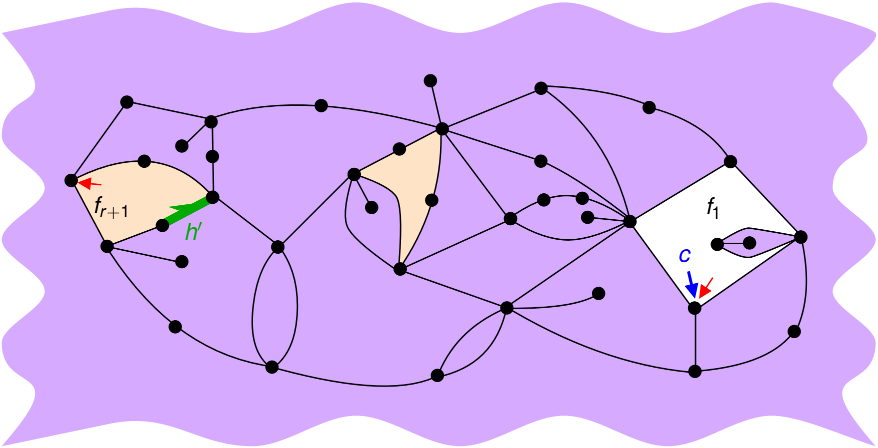
Consider the corner h_0 *delimited by* h *and its successor in* f_1.

Conversely



Consider the rightmost geodesic from h_0 toward c'.

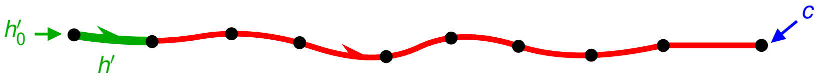
Conversely



Slit! Slide! Sew! And mark h' and c.

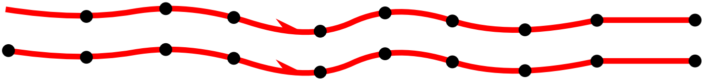
Proof

We need to see that the leftmost geodesic from h'_0 toward c becomes the rightmost geodesic from h_0 toward c'.



Proof

We need to see that the leftmost geodesic from h_0' toward c becomes the rightmost geodesic from h_0 toward c'.



Proof

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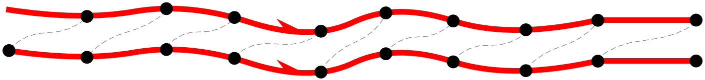

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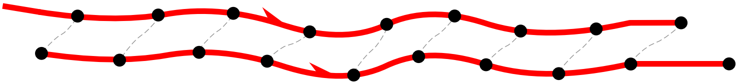

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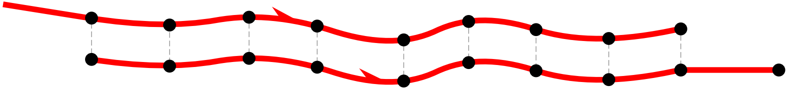

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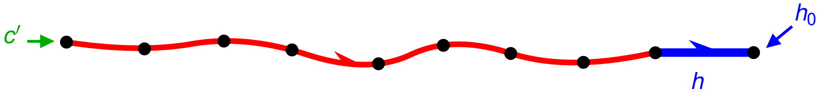

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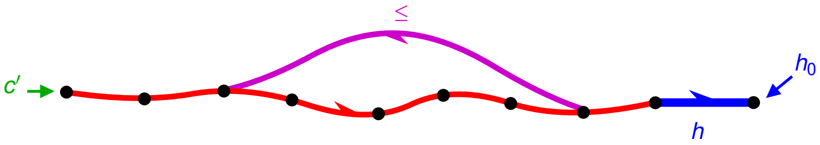

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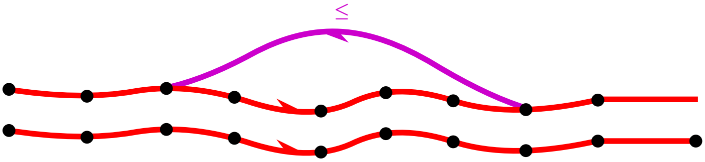

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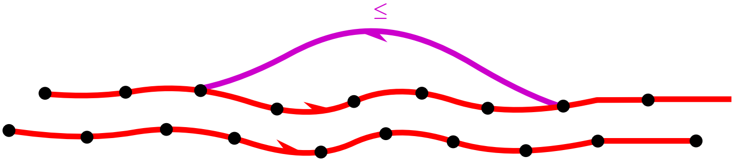

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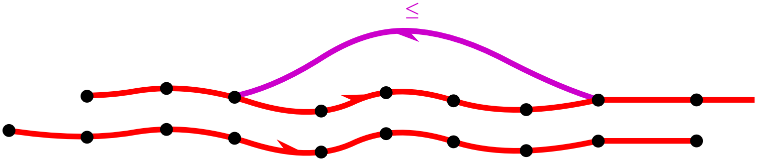

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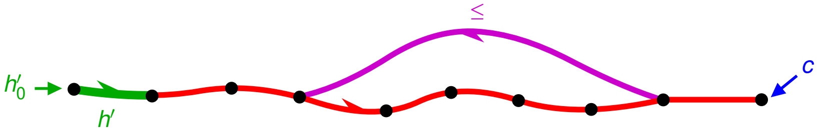

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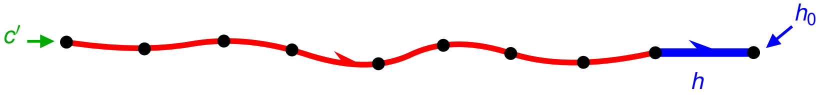

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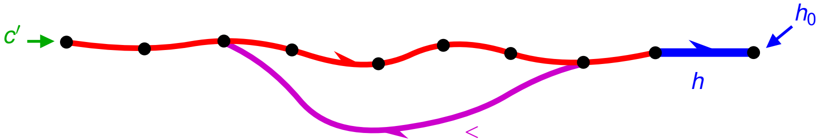

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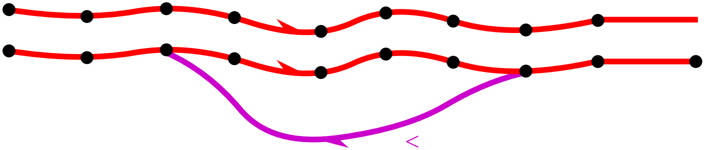

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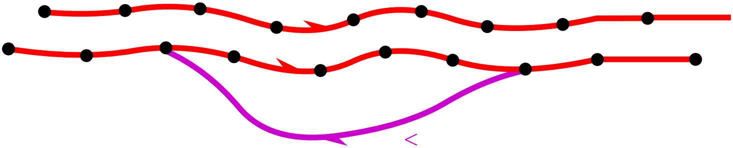

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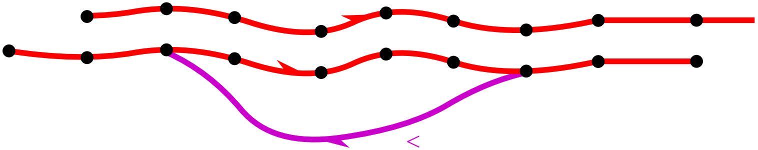

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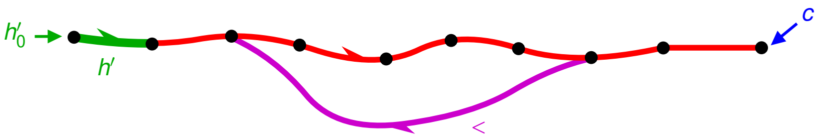

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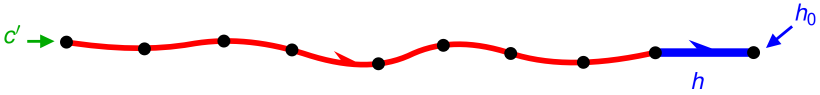

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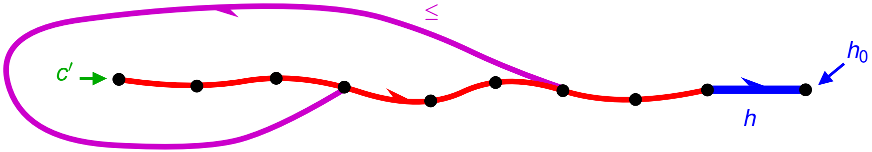

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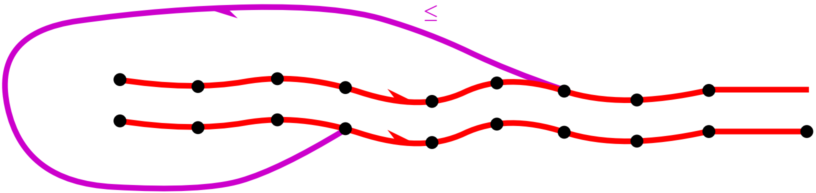

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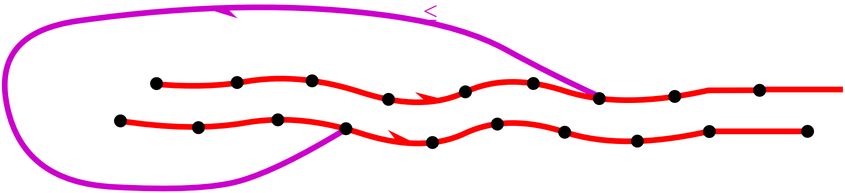

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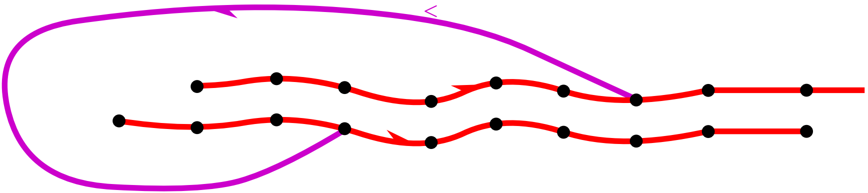

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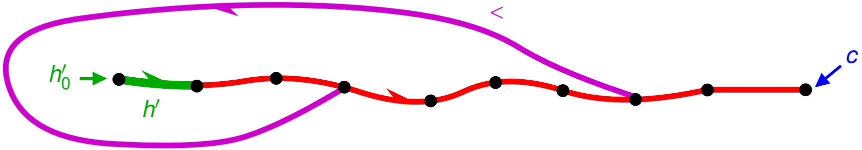

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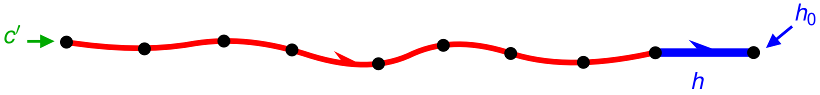
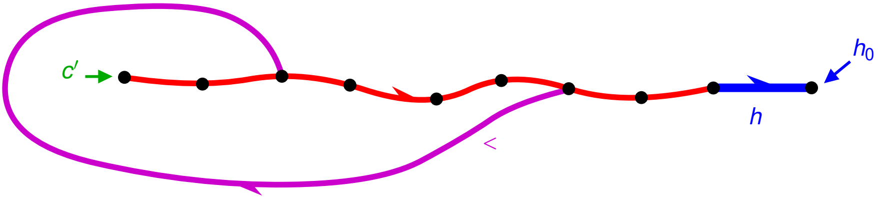


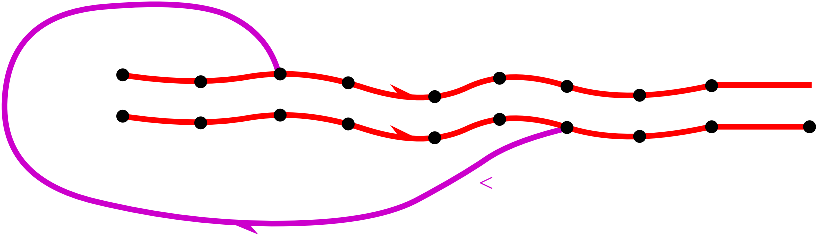
Proof

We need to see that the leftmost geodesic from h'_0 toward c becomes the rightmost geodesic from h_0 toward c'.



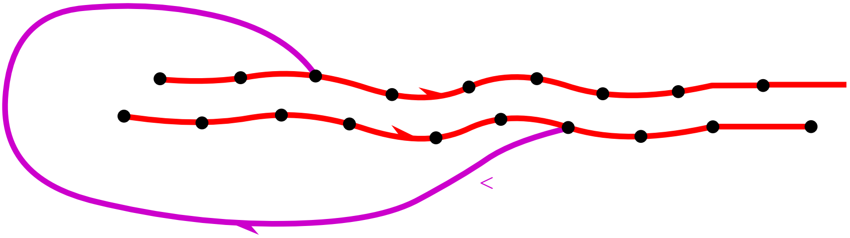
Proof

We need to see that the leftmost geodesic from h'_0 toward c becomes the rightmost geodesic from h_0 toward c'.



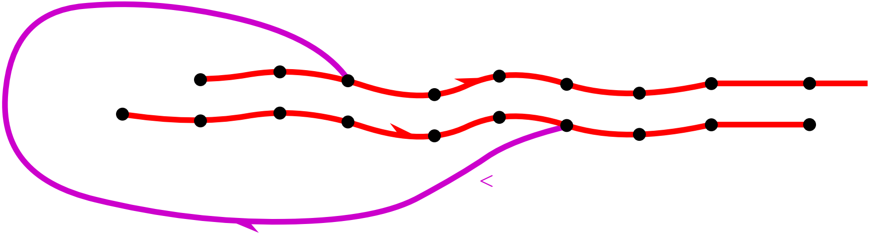
Proof

We need to see that the leftmost geodesic from h'_0 toward c becomes the rightmost geodesic from h_0 toward c'.



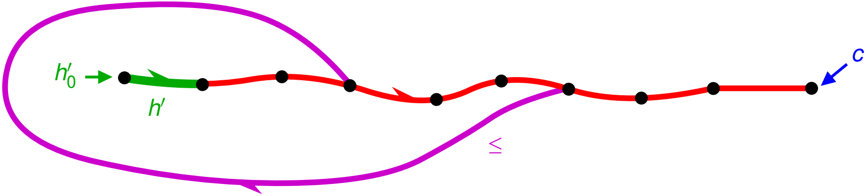
Proof

We need to see that the leftmost geodesic from h_0' toward c becomes the rightmost geodesic from h_0 toward c'.



Proof

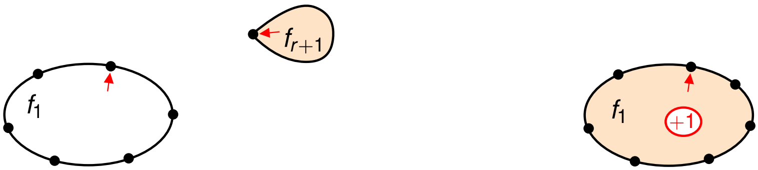
We need to see that the leftmost geodesic from h'_0 toward c becomes the rightmost geodesic from h_0 toward c'.



Transferring from a face of degree 1

$\boldsymbol{a} = (a_1, \dots, a_r, 1)$ $\tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \dots, a_r)$

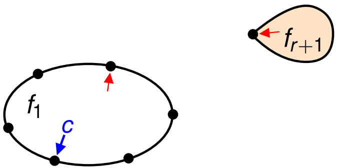
$$(a_1 + 1)\, M(\boldsymbol{a}) = \left\lfloor \tilde{a}_1/2 \right\rfloor \quad V(\tilde{\boldsymbol{a}})\ M(\tilde{\boldsymbol{a}})$$

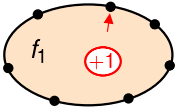


Transferring from a face of degree 1

$\boldsymbol{a} = (a_1, \dots, a_r, 1)$ $\qquad$ $\tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \dots, a_r)$

$$\underbrace{(a_1 + 1)}_{\substack{\text{corner} \\ c \text{ in } f_1}} M(\boldsymbol{a}) = \left\lfloor \tilde{a}_1/2 \right\rfloor \quad V(\tilde{\boldsymbol{a}})\ M(\tilde{\boldsymbol{a}})$$





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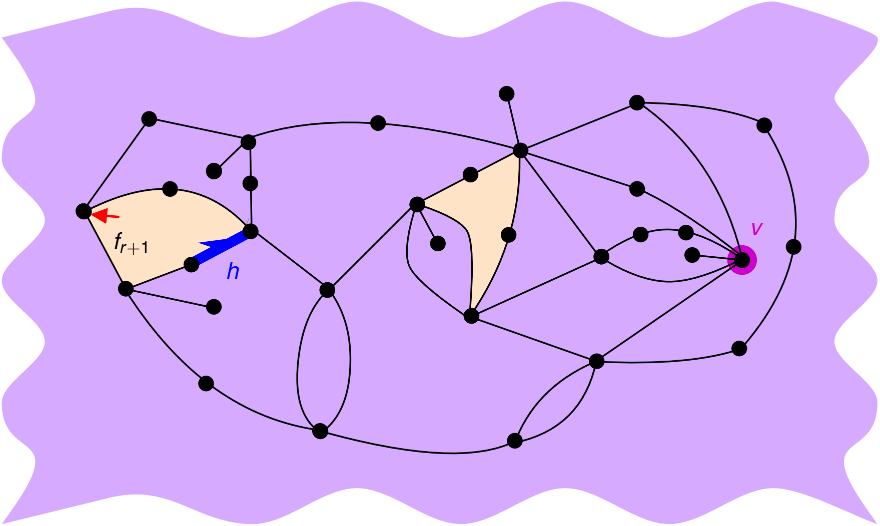
Transferring from a face of degree 1

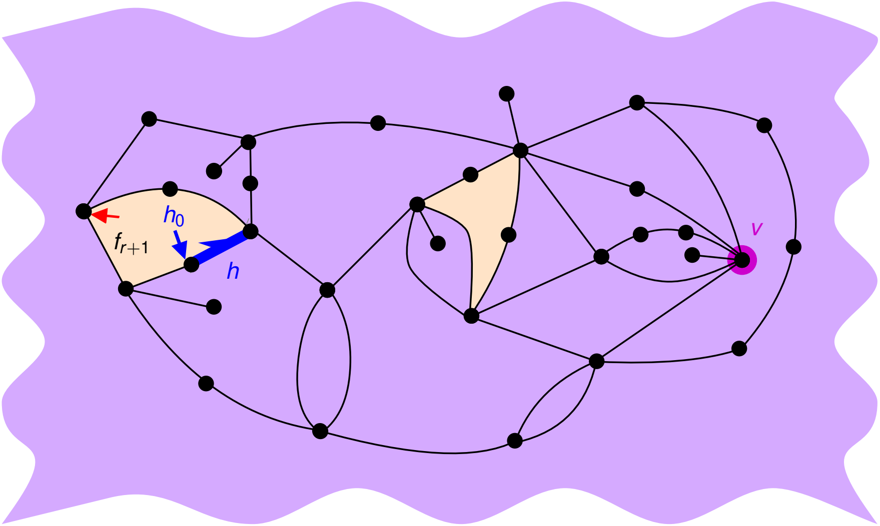
$\boldsymbol{a} = (a_1, \ldots, a_r, 1)$ $\qquad$ $\tilde{\boldsymbol{a}} := (a_1 + 1, a_2, \ldots, a_r)$

$$\underbrace{(a_1+1)}_{\substack{\text{corner} \\ c \text{ in } f_1}} M(\boldsymbol{a}) = \underbrace{\lfloor \tilde{a}_1/2 \rfloor}_{\substack{\text{half-edge } h \\ \text{of } f_1 \text{ toward } c'}} \underbrace{V(\tilde{\boldsymbol{a}})}_{\text{vertex } v} M(\tilde{\boldsymbol{a}})$$

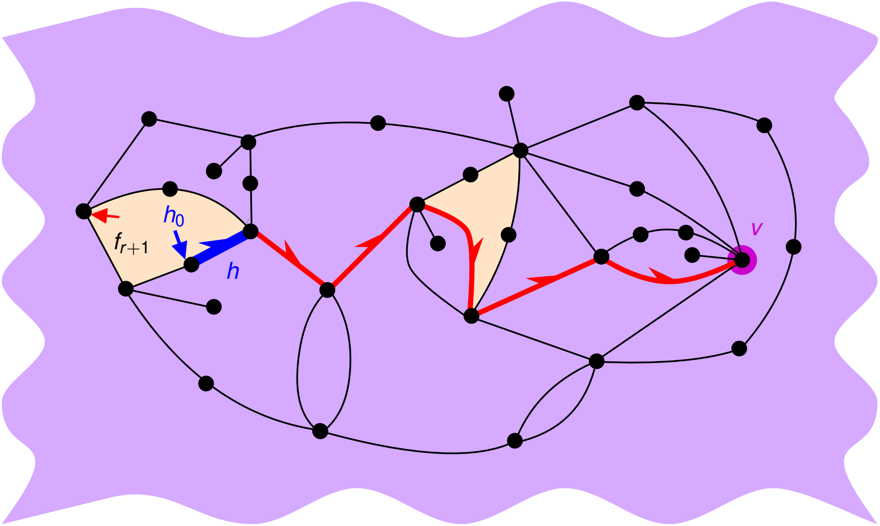
Transferring from a face of degree 1



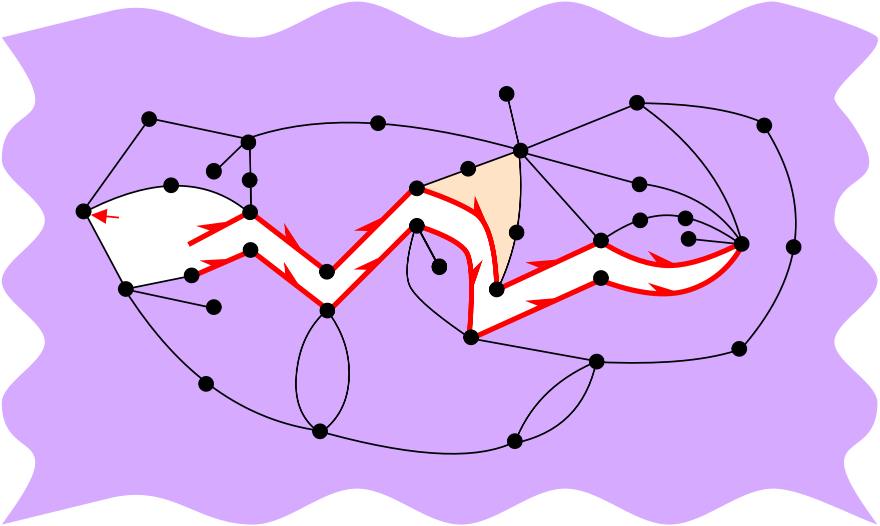
Transferring from a face of degree 1



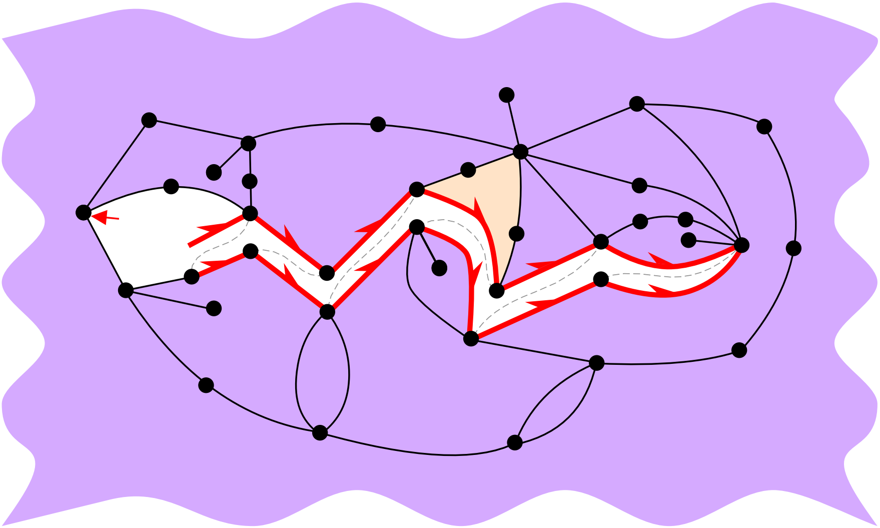
Transferring from a face of degree 1



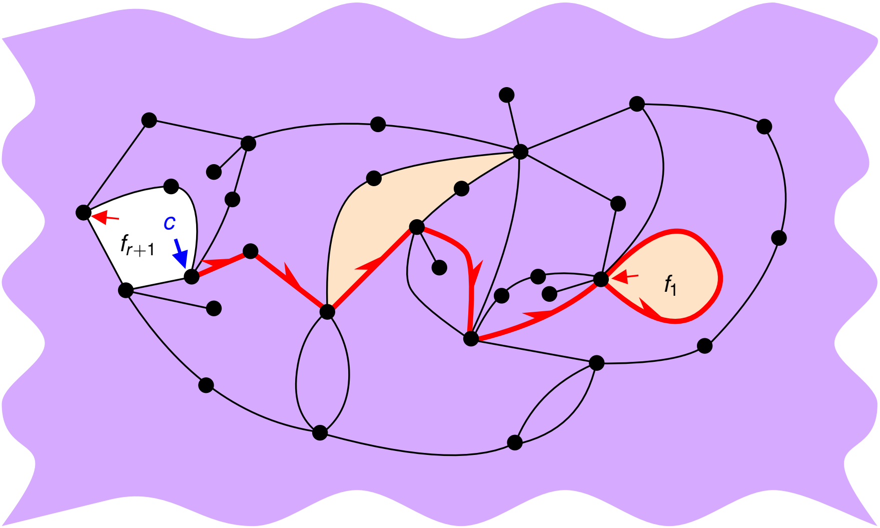
Transferring from a face of degree 1



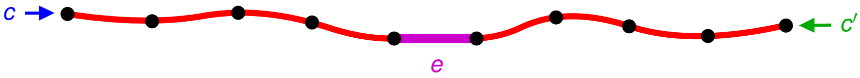
Transferring from a face of degree 1



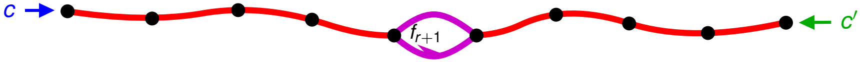
Transferring from a face of degree 1



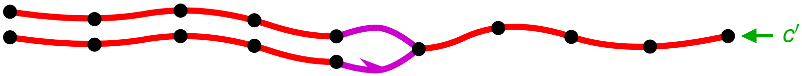
Decomposition into transfer bijections



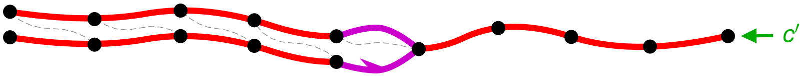
Decomposition into transfer bijections



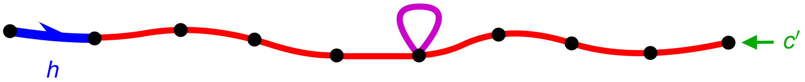
Decomposition into transfer bijections



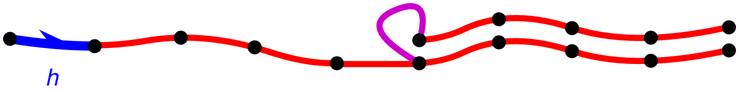
Decomposition into transfer bijections



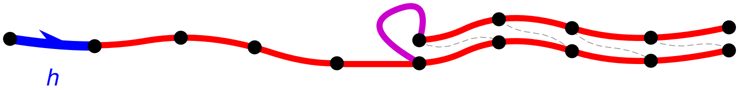
Decomposition into transfer bijections



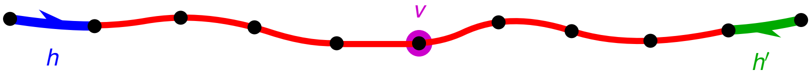
Decomposition into transfer bijections



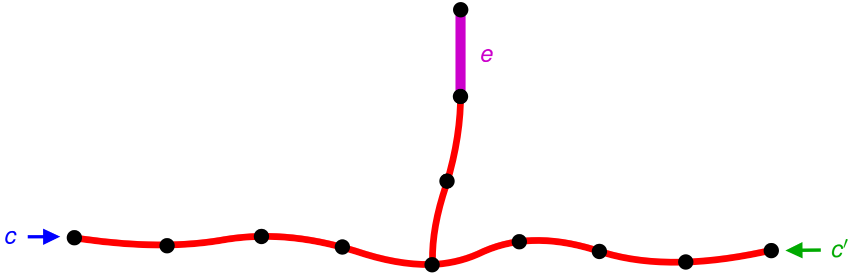
Decomposition into transfer bijections



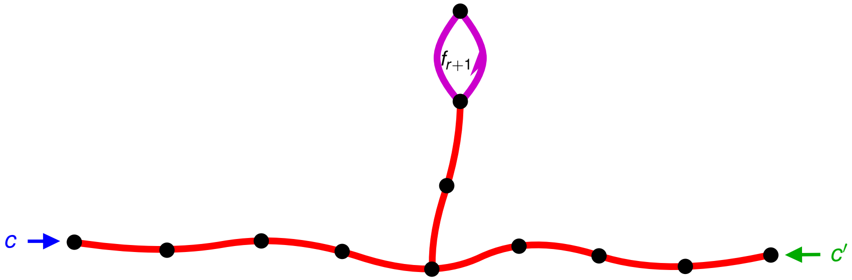
Decomposition into transfer bijections



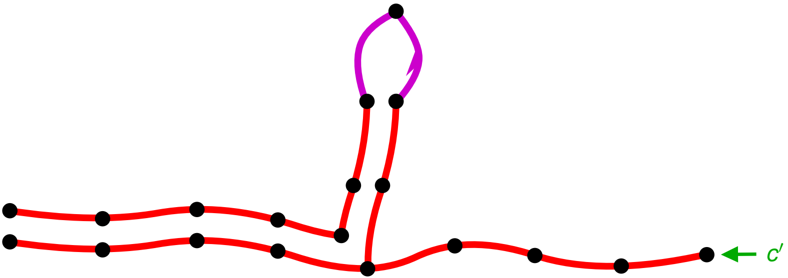
Decomposition into transfer bijections



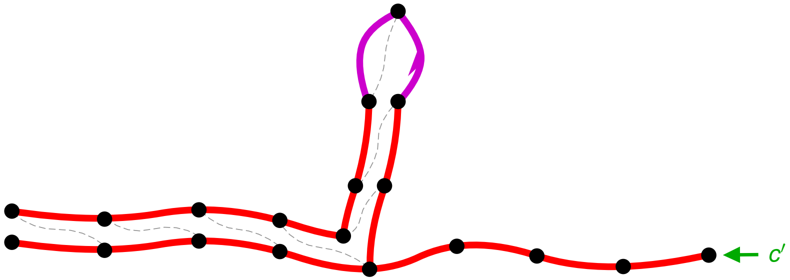
Decomposition into transfer bijections



Decomposition into transfer bijections



Decomposition into transfer bijections



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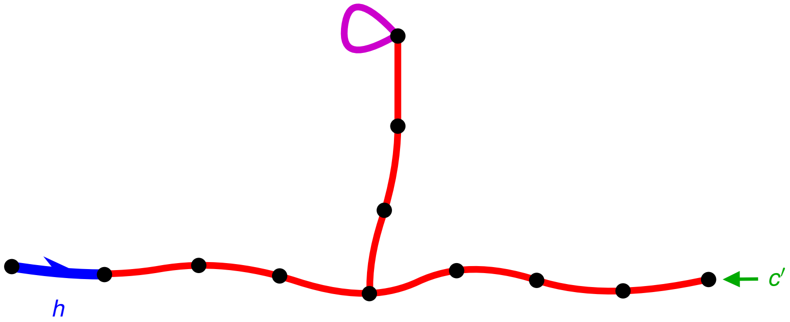

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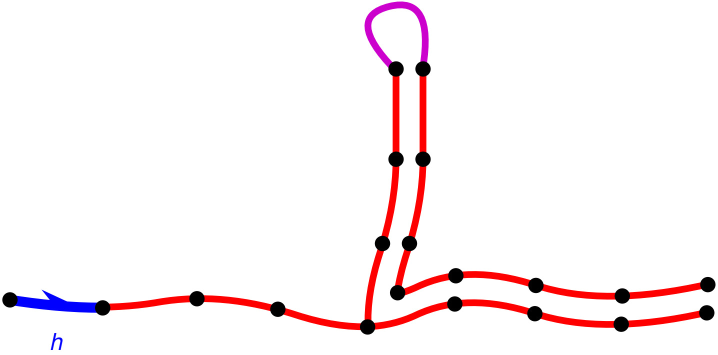
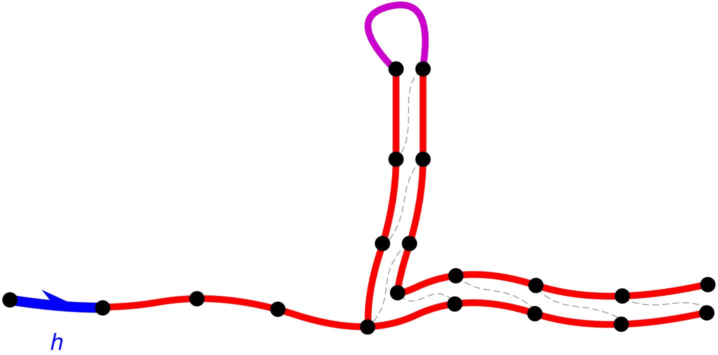
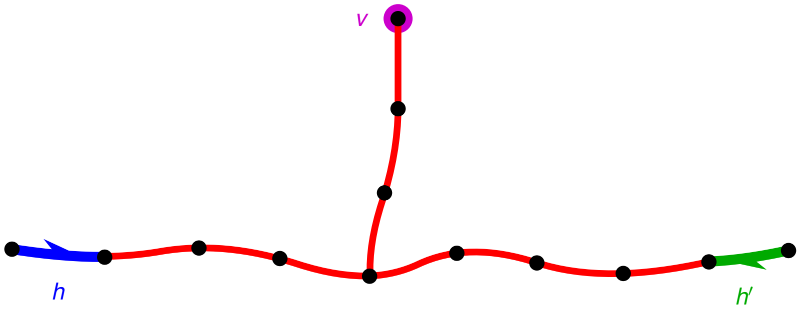


Decomposition into transfer bijections



Decomposition into transfer bijections



Last question

$\boldsymbol{a} = (a_1, \dots, a_r)$ $\qquad \tilde{\boldsymbol{a}} = (a_1 + 2, a_2, \dots, a_r)$

$$(a_1+1)\,(a_1+2)\,E(\boldsymbol{a})\,M(\boldsymbol{a}) = \lfloor \tilde{a}_1/2 \rfloor \lfloor (\tilde{a}_1 - 1)/2 \rfloor\, V(\tilde{\boldsymbol{a}})\,M(\tilde{\boldsymbol{a}})$$

$\boldsymbol{a} = (a_1, \dots, a_r)$ $\qquad \tilde{\boldsymbol{a}} = (a_1 + 1, a_2 + 1, a_3, \dots, a_r)$

$$(a_1+1)\,(a_2+1)\,E(\boldsymbol{a})\,M(\boldsymbol{a}) = \lfloor \tilde{a}_1/2 \rfloor \lfloor \tilde{a}_2/2 \rfloor\, V(\tilde{\boldsymbol{a}})\,M(\tilde{\boldsymbol{a}})$$

$\boldsymbol{a} = (a_1, \dots, a_{r+1})$ $\qquad \tilde{\boldsymbol{a}} = (a_1 + 1, a_2, \dots, a_r, a_{r+1} - 1)$

$$(a_1+1)\,\lfloor a_{r+1}/2 \rfloor\,M(\boldsymbol{a}) = \lfloor \tilde{a}_1/2 \rfloor\,(\tilde{a}_{r+1}+1)\,M(\tilde{\boldsymbol{a}})$$

Still valid for quasibipartite maps! But...

- The edge may be parallel to the corners (and may be a loop).
- Not so clear to interpret the green terms.
- In an even degree face, 0 or 2 half-edges are parallel to a corner.
- No longer bip. or quasibip. when decomposing as transfer bij.

THANK
YOU