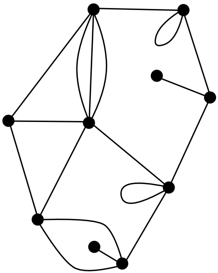
Scaling Limit of Arbitrary Genus Random Maps

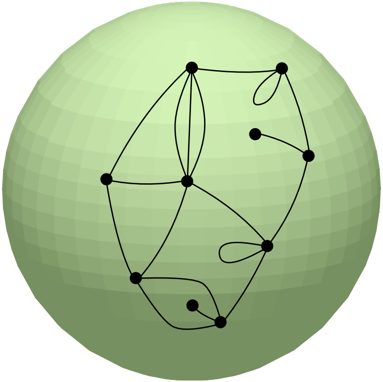
Jérémie BETTINELLI

UNIVERSITÉ PARIS SUD

February 1, 2012

Planar maps





planar map: finite connected graph embedded in the sphere

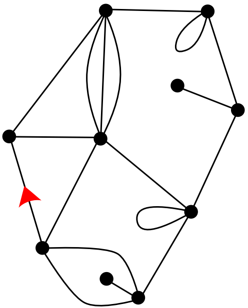
faces: connected components of the complement

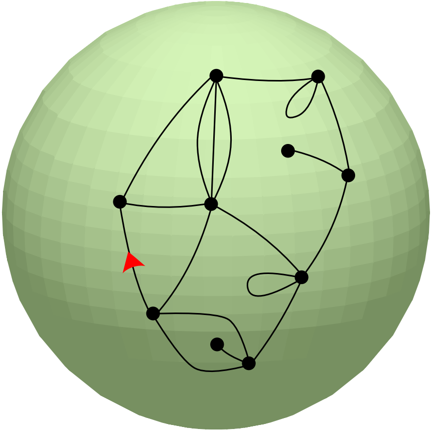
Example of planar map

faces:
countries and
bodies of water

connected graph
no “enclaves”

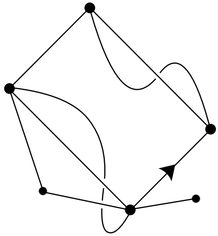
Rooted maps

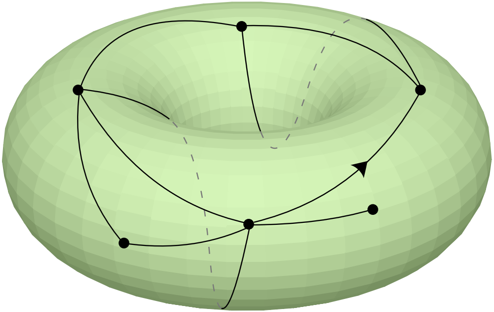




rooted map: map with one distinguished oriented edge

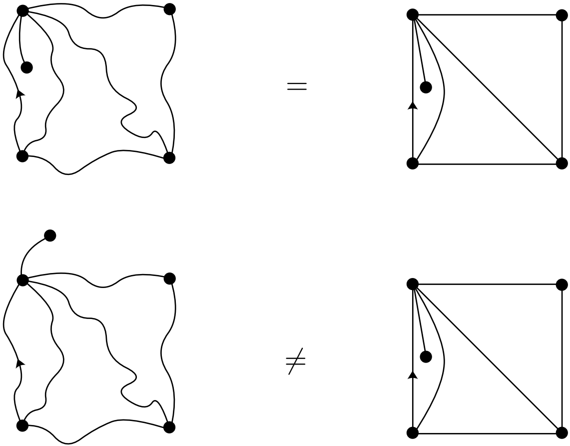
Genus g-maps



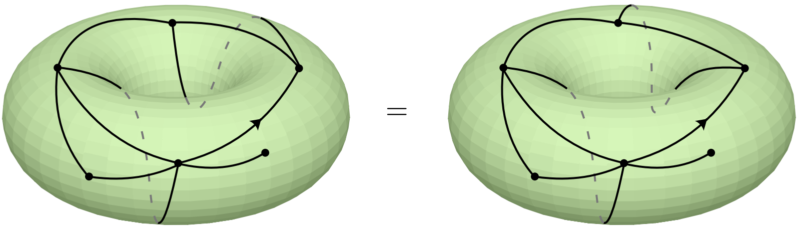


Genus g-map: graph embedded in the g-torus, in such a way that the faces are homeomorphic to disks

Edge deformation



More complicated deformation



maps are defined up to direct homeomorphism of the underlying surface

"What does a large random map look like?"

“What does a large random map look like?”

point of view

We see a map $\mathfrak{m}$ as a discrete metric space $(V(\mathfrak{m}), d_{\mathfrak{m}})$:

- $V(\mathfrak{m})$: vertex set of $\mathfrak{m}$
- $d_{\mathfrak{m}}(u, v)$: smallest $k \geq 0$ such that there exists a path with k edges linking u to v

"What does a large random map look like?"

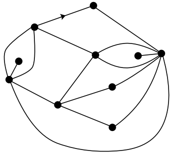
point of view

We see a map $\mathfrak{m}$ as a discrete metric space $(V(\mathfrak{m}), d_{\mathfrak{m}})$:

- $V(\mathfrak{m})$: vertex set of $\mathfrak{m}$
- $d_{\mathfrak{m}}(u, v)$: smallest $k \geq 0$ such that there exists a path with k edges linking u to v

randomness

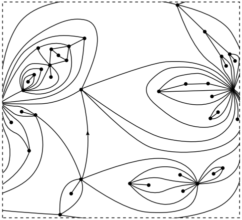
We fix the genus g and choose $\mathfrak{q}_n$ uniformly at random among all genus g (bipartite) quadrangulations with n faces



$$\left(V(\mathfrak{q}_n), d_{\mathfrak{q}_n}\right) \xrightarrow[n\to\infty]{} ?$$

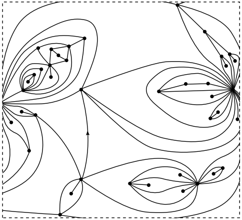
Short history ($g = 0$)

Angel & Schramm ('02)
local limit
(random planar triangulations)



Short history ($g = 0$)

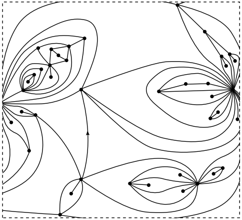
Angel & Schramm ('02)
local limit
(random planar triangulations)



Chassaing & Schaeffer ('04)
u_n and v_n uniform in $V(\mathfrak{q}_n)$
$d_{\mathfrak{q}_n}(u_n, v_n) \sim n^{1/4}$

Short history ($g = 0$)

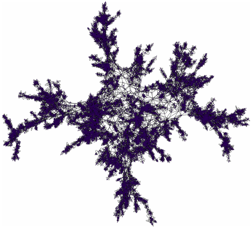
Angel & Schramm ('02)
local limit
(random planar triangulations)



Chassaing & Schaeffer ('04)
u_n and v_n uniform in $V(\mathfrak{q}_n)$
$d_{\mathfrak{q}_n}(u_n, v_n) \sim n^{1/4}$

Marckert & Mokkadem ('06)
Le Gall ('07)
scaling limit

$$\left(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n}\right) \xrightarrow[n\to\infty]{} ?$$



realized by J.-F. Marckert

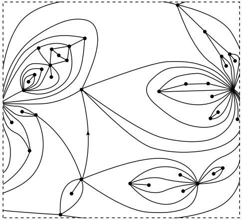
Short history ($g = 0$)

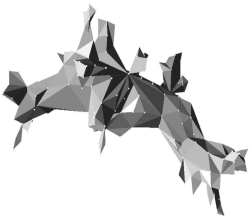
Angel & Schramm ('02)
local limit
(random planar triangulations)

Chassaing & Schaeffer ('04)
u_n and v_n uniform in $V(\mathfrak{q}_n)$
$d_{\mathfrak{q}_n}(u_n, v_n) \sim n^{1/4}$

Marckert & Mokkadem ('06)
Le Gall ('07)
scaling limit

$$\left(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n}\right) \xrightarrow[n\to\infty]{} ?$$





realized by G. Chapuy

The Brownian map ($g = 0$)

- $\mathfrak{q}_n$ uniform among planar quadrangulations with n faces

Theorem (Le Gall '11, Miermont '11)

*The metric space $\left(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n}\right)$ converges in distribution toward a random metric space, called **the Brownian map** for the Gromov–Hausdorff topology.*

The Brownian map ($g = 0$)

- $\mathfrak{q}_n$ uniform among planar quadrangulations with n faces

Theorem (Le Gall '11, Miermont '11)

*The metric space $(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n})$ converges in distribution toward a random metric space, called **the Brownian map** for the Gromov–Hausdorff topology.*

Definition (Convergence for the G–H topology)

A sequence $(\mathcal{X}_n)$ of compact metric spaces **converges for the Gromov–Hausdorff topology** toward a metric space $\mathcal{X}$ if there exist isometric embeddings $\varphi_n : \mathcal{X}_n \to \mathcal{Z}$ and $\varphi : \mathcal{X} \to \mathcal{Z}$ into a common metric space $\mathcal{Z}$ such that $\varphi_n(\mathcal{X}_n)$ converges toward $\varphi(\mathcal{X})$ for the Hausdorff topology.

Two properties of the Brownian map

Theorem (Le Gall '07)

The Hausdorff dimension of the Brownian map is almost surely equal to 4.

Theorem (Le Gall & Paulin '08, Miermont '08)

The Brownian map is almost surely homeomorphic to the 2-dimensional sphere.

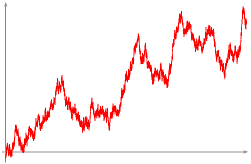
Two properties of the Brownian map

Theorem (Le Gall '07)

The Hausdorff dimension of the Brownian map is almost surely equal to 4.

Theorem (Le Gall & Paulin '08, Miermont '08)

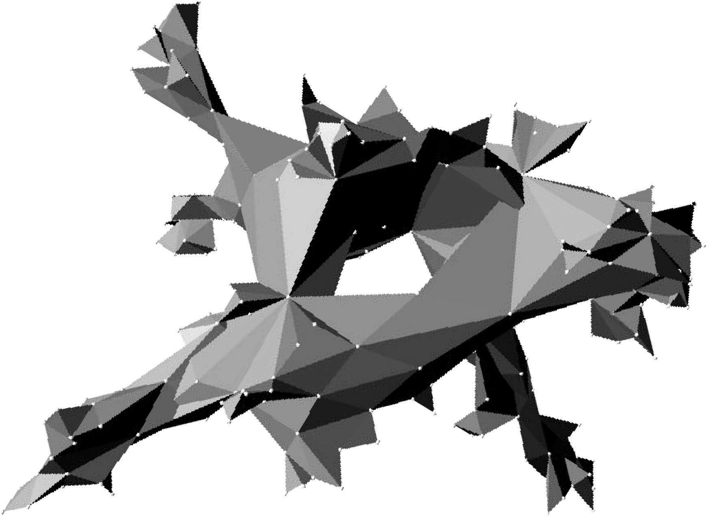
The Brownian map is almost surely homeomorphic to the 2-dimensional sphere.



graph of a Brownian motion

- dimension: $3/2$
- homeomorphic to $\mathbb{R}$

Positive (fixed) genus $g \geq 1$



realized by G. Chapuy

Positive (fixed) genus $g \geq 1$

- $\mathfrak{q}_n$ uniform among bipartite genus g quadrangulations with n faces

Theorem (B. '10)

The metric space $(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n})$ converges weakly, up to extraction, toward a random metric space $(\mathfrak{q}_\infty, d_\infty)$.

This extends results of G. Chapuy who showed in particular that typical distances were, as in the planar case, of order $n^{1/4}$

Positive (fixed) genus $g \geq 1$

- $\mathfrak{q}_n$ uniform among bipartite genus g quadrangulations with n faces

Theorem (B. '10)

The metric space $(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n})$ converges weakly, up to extraction, toward a random metric space $(\mathfrak{q}_\infty, d_\infty)$.

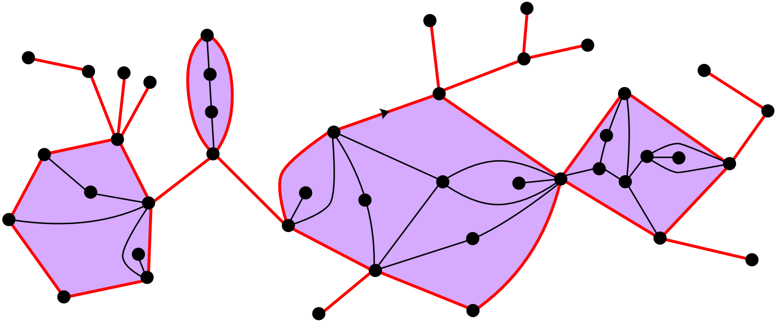
Theorem (B. '10)

The Hausdorff dimension of any possible $(\mathfrak{q}_\infty, d_\infty)$ from the previous theorem is almost surely equal to 4.

Theorem (B. '11)

Any possible $(\mathfrak{q}_\infty, d_\infty)$ from the previous theorem is almost surely homeomorphic to the genus g-torus.

Planar quadrangulations with a boundary



quadrangulation with a boundary: planar map whose faces are all quadrangles except possibly the external face

The boundary is not required to be a simple curve

Scaling limit: general case

- $\mathfrak{q}_n$ uniform among quadrangulations with a boundary having n faces and $2\sigma_n$ half-edges on the boundary
- $\sigma_n/\sqrt{2n} \to \sigma \in]0, \infty[$

Theorem (B. '11)

The metric space $(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n})$ converges weakly, up to extraction, toward a metric space $(\mathfrak{q}^\sigma, d^\sigma)$ of dimension 4 a.s.

Theorem (B. '11)

Any possible $(\mathfrak{q}^\sigma, d^\sigma)$ from the previous theorem is almost surely homeomorphic to the 2-dimensional disk. Moreover, its boundary is of dimension 2 a.s.

Scaling limit: degenerate cases

- $\mathfrak{q}_n$ uniform among quadrangulations with a boundary having n faces and $2\sigma_n$ half-edges on the boundary
- $\sigma_n/\sqrt{2n} \to 0$

Theorem (B. '11)

The metric space $\left(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n}\right)$ converges weakly toward the Brownian map.

Scaling limit: degenerate cases

- $\mathfrak{q}_n$ uniform among quadrangulations with a boundary having n faces and $2\sigma_n$ half-edges on the boundary
- $\sigma_n/\sqrt{2n} \to 0$

Theorem (B. '11)

The metric space $\left(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n}\right)$ converges weakly toward the Brownian map.

- $\sigma_n/\sqrt{2n} \to \infty$

Theorem (B. '11)

The metric space $\left(V(\mathfrak{q}_n), (2\sigma_n)^{-1/2} d_{\mathfrak{q}_n}\right)$ converges weakly toward the Continuum Random Tree.

Scaling limit: degenerate cases

- $\sigma_n/\sqrt{2n} \to 0$

Theorem (B. '11)

The metric space $\left(V(\mathfrak{q}_n), n^{-1/4} d_{\mathfrak{q}_n}\right)$ converges weakly toward the Brownian map.

- $\sigma_n/\sqrt{2n} \to \infty$

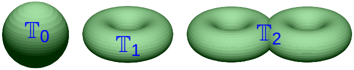
Theorem (B. '11)

The metric space $\left(V(\mathfrak{q}_n), (2\sigma_n)^{-1/2} d_{\mathfrak{q}_n}\right)$ converges weakly toward the Continuum Random Tree.

These results agree with the recent work of J. Bouttier & E. Guitter who observed these three distinct regimes in a context of distance statistics

Problem

- $\mathbb{T}_0$: 2-dimensional sphere
- $\mathbb{T}_g$: torus with g holes ($g \geq 1$)

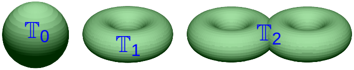


Question

Consider a sequence $(\mathcal{X}_n)$ of compact metric spaces all homeomorphic to $\mathbb{T}_g$ (g is fixed) that converges toward a metric space $\mathcal{X}$. Is $\mathcal{X}$ homeomorphic to $\mathbb{T}_g$?

Problem

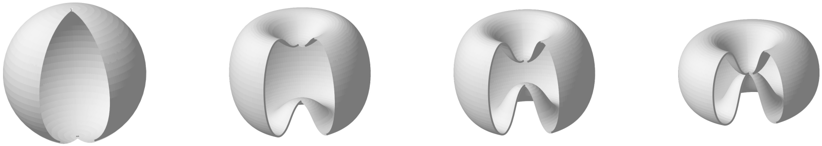
- $\mathbb{T}_0$: 2-dimensional sphere
- $\mathbb{T}_g$: torus with g holes ($g \geq 1$)



Question

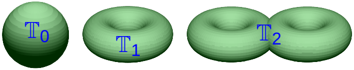
Consider a sequence $(\mathcal{X}_n)$ of compact metric spaces all homeomorphic to $\mathbb{T}_g$ (g is fixed) that converges toward a metric space $\mathcal{X}$. Is $\mathcal{X}$ homeomorphic to $\mathbb{T}_g$?

No!



Problem

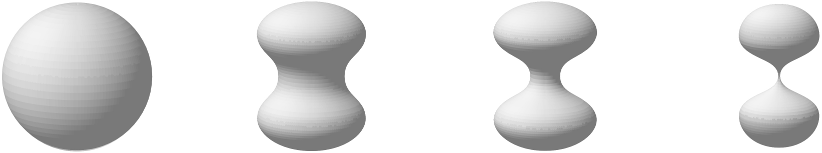
- $\mathbb{T}_0$: 2-dimensional sphere
- $\mathbb{T}_g$: torus with g holes ($g \geq 1$)



Question

Consider a sequence $(\mathcal{X}_n)$ of compact metric spaces all homeomorphic to $\mathbb{T}_g$ (g is fixed) that converges toward a metric space $\mathcal{X}$. Is $\mathcal{X}$ homeomorphic to $\mathbb{T}_g$?

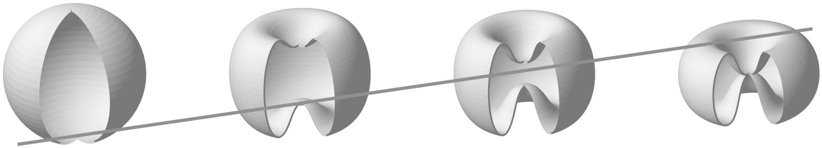
No!



0-regularity

Definition (0-regularity)

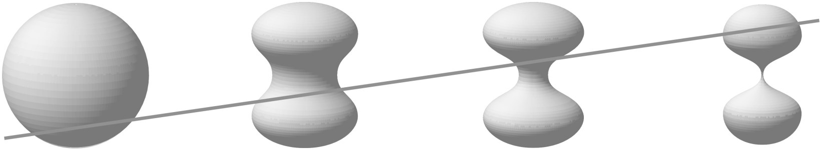
A sequence $(\mathcal{X}_n)_n$ of compact metric spaces is **0-regular** if for every $\varepsilon > 0$, there exists $\eta > 0$ such that, for n large enough, any two points lying at distance less than η are in a connected subset of $\mathcal{X}_n$ with diameter smaller than ε.



1-regularity

Definition (1-regularity)

A sequence $(\mathcal{X}_n)_n$ of compact metric spaces is **1-regular** if for every $\varepsilon > 0$, there exists $\eta > 0$ such that, for n large enough, every loop of diameter less than η in $\mathcal{X}_n$ is homotopic to 0 in its ε-neighborhood.



Convergence of regular sequences

Theorem (Begle '44)

Let $(\mathcal{X}_n)_n$ be a sequence of compact metric spaces all homeomorphic to $\mathbb{T}_g$ such that $\mathcal{X}_n \xrightarrow{GH} \mathcal{X}$. Suppose that $(\mathcal{X}_n)_n$ is both 0 and 1-regular.
Then $\mathcal{X}$ is either homeomorphic to $\mathbb{T}_g$ or reduced to a single point (this case can only happen when $g = 0$).

Convergence of regular sequences

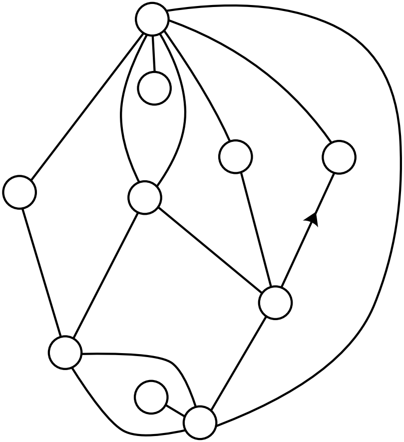
Theorem (Begle '44)

Let $(\mathcal{X}_n)_n$ be a sequence of compact metric spaces all homeomorphic to $\mathbb{T}_g$ such that $\mathcal{X}_n \xrightarrow{GH} \mathcal{X}$. Suppose that $(\mathcal{X}_n)_n$ is both 0 and 1-regular.
Then $\mathcal{X}$ is either homeomorphic to $\mathbb{T}_g$ or reduced to a single point (this case can only happen when $g = 0$).

Theorem (Whyburn '35)

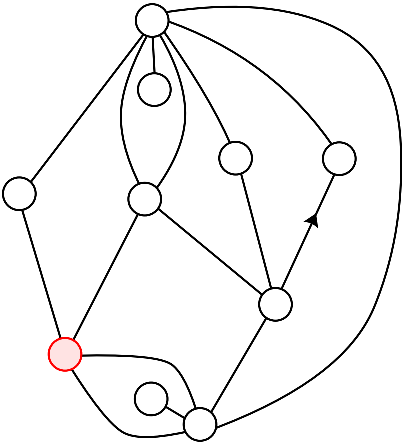
Let $(\mathcal{X}_n)_n$ be a sequence of compact metric spaces all homeomorphic to the 2- dimensional disc $\mathbb{D}_2$ such that $\mathcal{X}_n \xrightarrow{GH} \mathcal{X}$. Suppose that $(\mathcal{X}_n)_n$ is both 0 and 1-regular, and that $(\partial\mathcal{X}_n)_n$ is 0-regular.
Then $\mathcal{X}$ is either homeomorphic to $\mathbb{D}_2$ or reduced to a point.

Cori–Vauquelin–Schaeffer's bijection



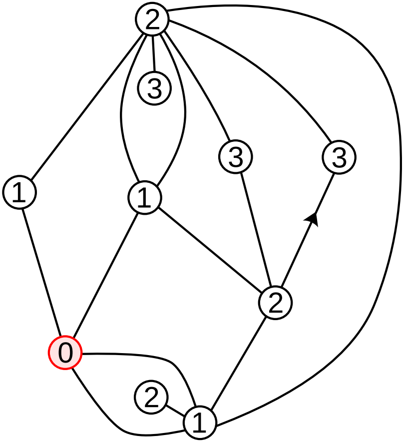
- we start with a planar quadrangulation

Cori–Vauquelin–Schaeffer's bijection



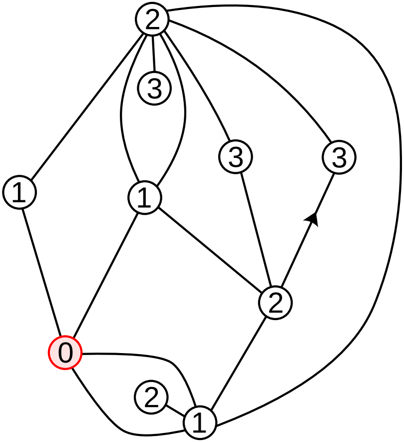
- we start with a planar quadrangulation
- we select a vertex

Cori–Vauquelin–Schaeffer's bijection

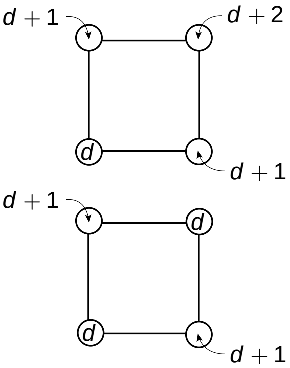


- we start with a planar quadrangulation
- we select a vertex ○
- we label each vertex with its distance to the selected vertex ○

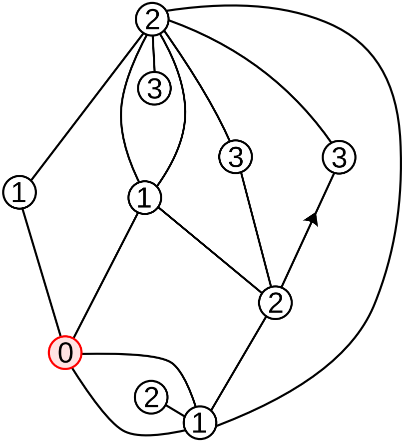
Cori–Vauquelin–Schaeffer's bijection



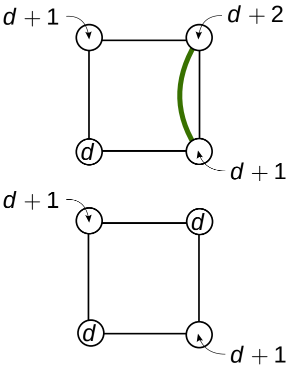
- we add a new edge on each face as follows:



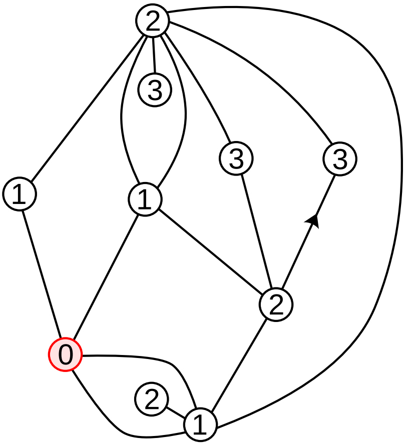
Cori–Vauquelin–Schaeffer's bijection



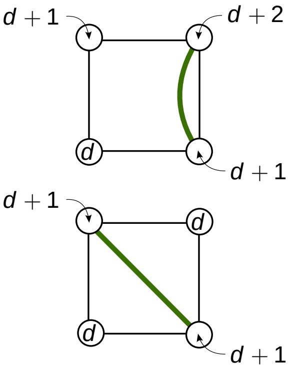
- we add a new edge on each face as follows:



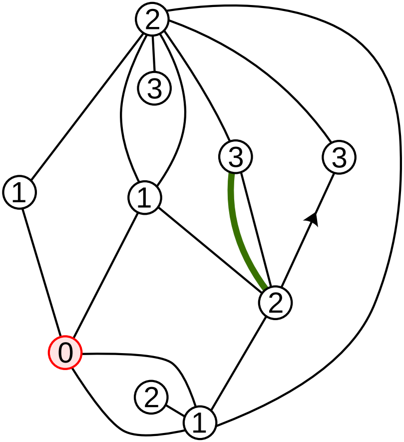
Cori–Vauquelin–Schaeffer's bijection



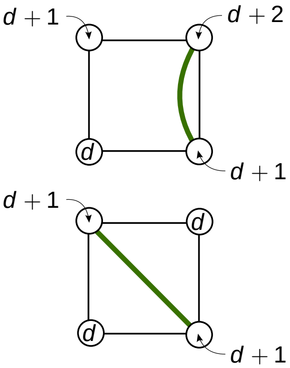
- we add a new edge on each face as follows:



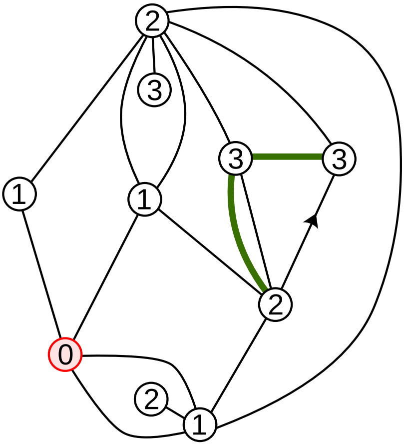
Cori–Vauquelin–Schaeffer's bijection



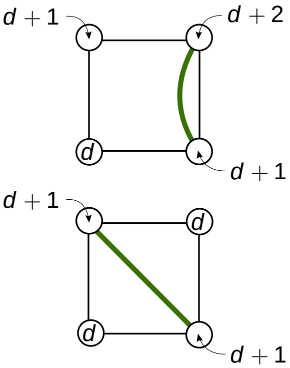
- we add a new edge on each face as follows:



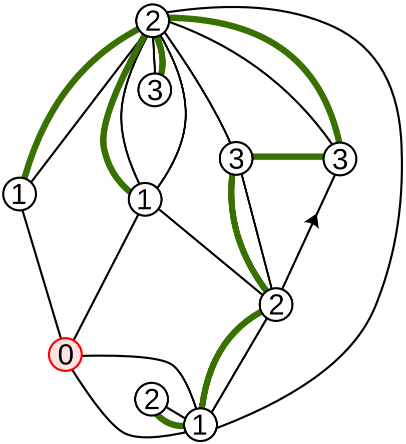
Cori–Vauquelin–Schaeffer's bijection



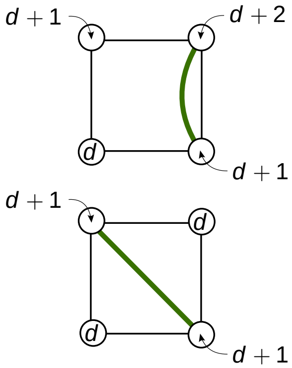
- we add a new edge on each face as follows:



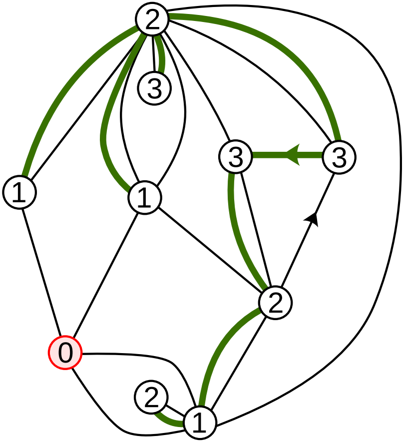
Cori–Vauquelin–Schaeffer's bijection



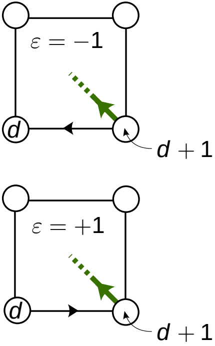
- we add a new edge on each face as follows:



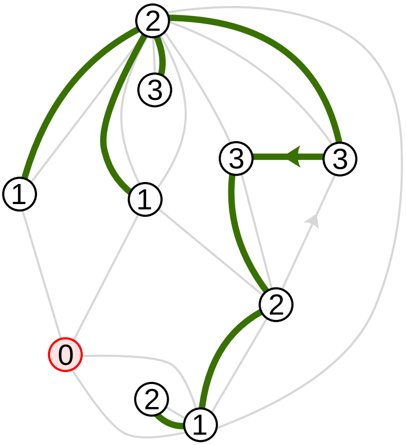
Cori–Vauquelin–Schaeffer's bijection



- we add a new root:

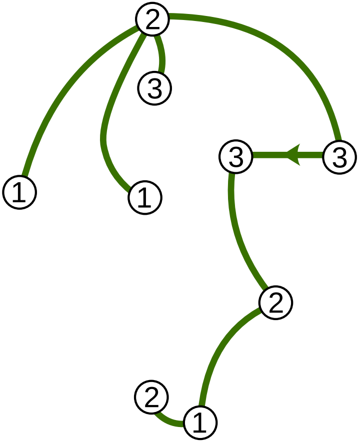


Cori–Vauquelin–Schaeffer's bijection



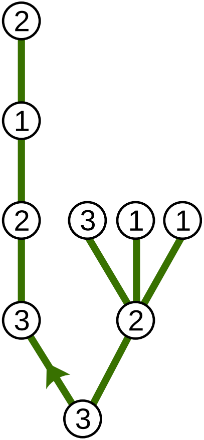
- we delete the old edges

Cori–Vauquelin–Schaeffer's bijection



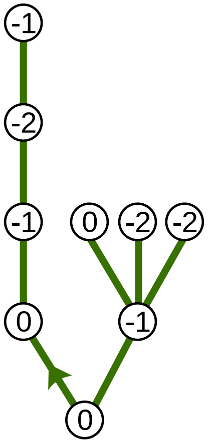
- we delete the old edges
- we delete the selected vertex ○

Cori–Vauquelin–Schaeffer's bijection



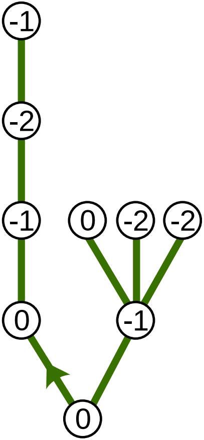
- we delete the old edges
- we delete the selected vertex

Cori–Vauquelin–Schaeffer's bijection



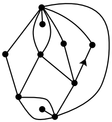
- we delete the old edges
- we delete the selected vertex ○
- we shift the labels in such a way that the root vertex has label 0

Cori–Vauquelin–Schaeffer's bijection



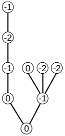
- we delete the old edges
- we delete the selected vertex ○
- we shift the labels in such a way that the root vertex has label 0
- we obtain a **well-labeled tree**

Coding maps with simpler objects



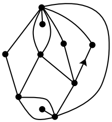
planar quad.

Cori–Vauquelin
–Schaeffer



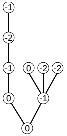
well-labeled tree

Coding maps with simpler objects

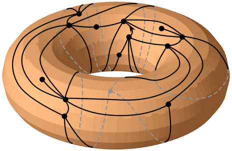


planar quad.

Cori–Vauquelin –Schaeffer ⟷

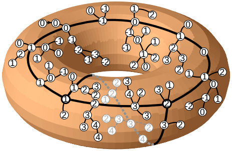


well-labeled tree



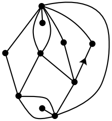
genus g bip. quad.

Chapuy–Marcus –Schaeffer ⟷



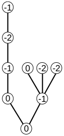
well-labeled g-tree

Coding maps with simpler objects

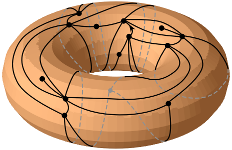


planar quad.

Cori–Vauquelin –Schaeffer ⟷

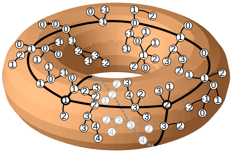


well-labeled tree

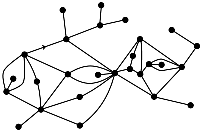


genus g bip. quad.

Chapuy–Marcus –Schaeffer ⟷

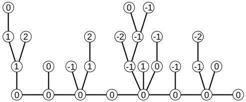


well-labeled g-tree



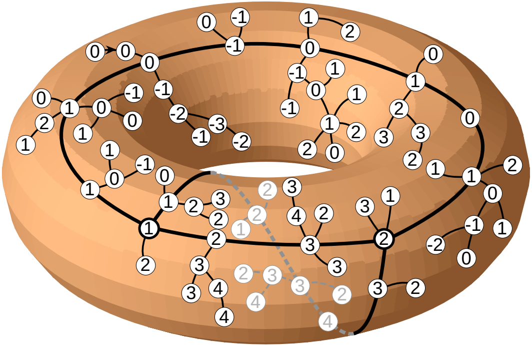
quad. with a bdry

Bouttier–Di Francesco –Guitter ⟷

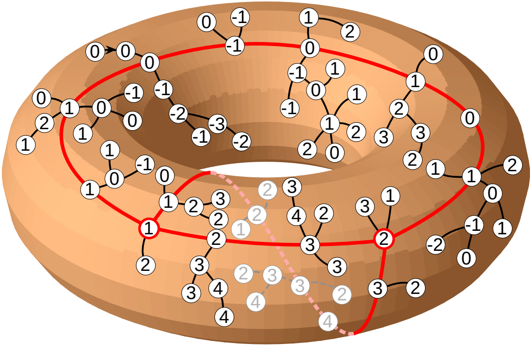


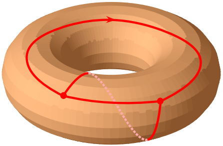
well-labeled forest (+ bridge)

Decomposition of a well-labeled *g*-tree



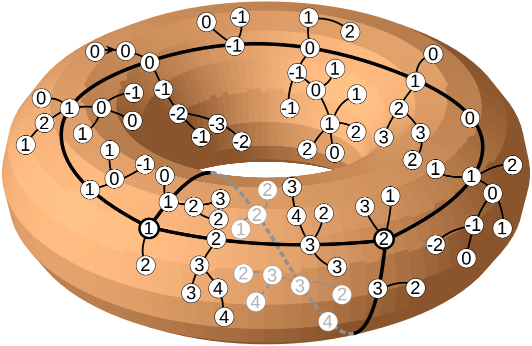
Decomposition of a well-labeled g-tree

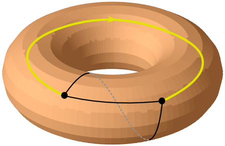




scheme $\mathfrak{s}$

Decomposition of a well-labeled g-tree

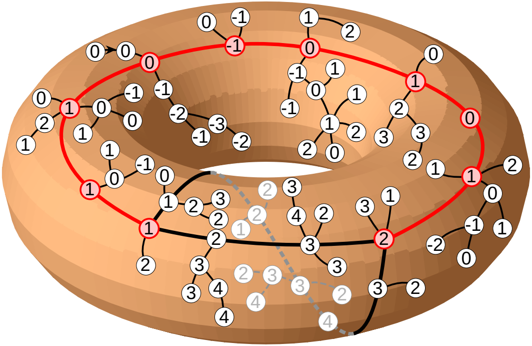


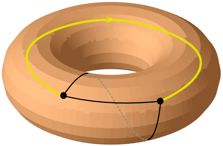


scheme $\mathfrak{s}$

With each edge of $\mathfrak{s}$, we associate:

Decomposition of a well-labeled g-tree



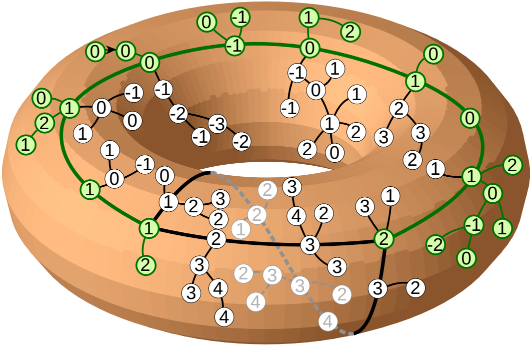


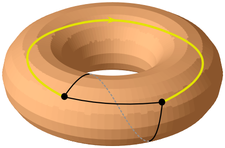
scheme $\mathfrak{s}$

With each edge of $\mathfrak{s}$, we associate:

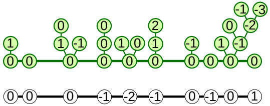
- a Motzkin bridge

Decomposition of a well-labeled g-tree





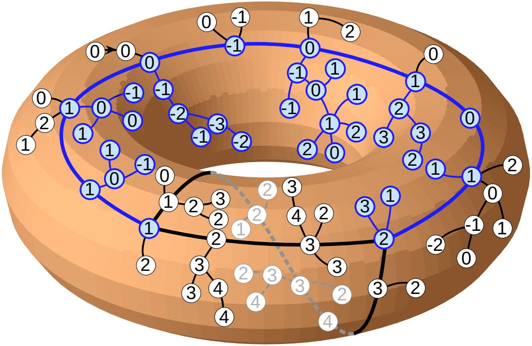
scheme $\mathfrak{s}$

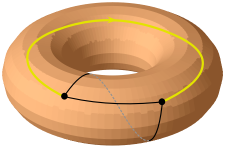


With each edge of $\mathfrak{s}$, we associate:

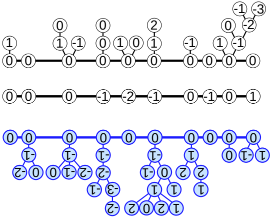
- a Motzkin bridge
- two well-labeled forests

Decomposition of a well-labeled g-tree





scheme $\mathfrak{s}$

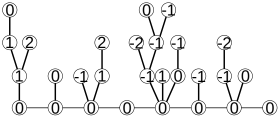


With each edge of $\mathfrak{s}$, we associate:

- a Motzkin bridge
- two well-labeled forests

Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

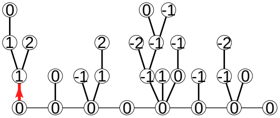


$(\mathfrak{f}, \mathfrak{l})$

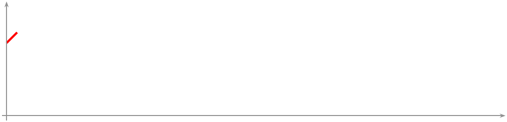
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



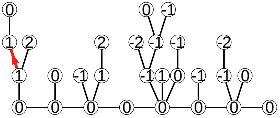
$(\mathfrak{f}, \mathfrak{l})$



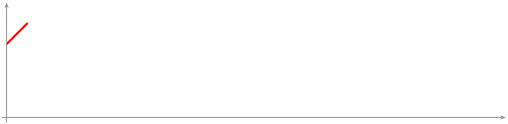
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



$(\mathfrak{f}, \mathfrak{l})$



Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

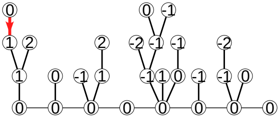
- $C^{\mathfrak{f}}$: contour function

$(\mathfrak{f}, \mathfrak{l})$

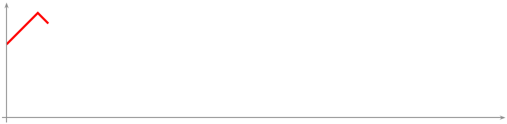
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



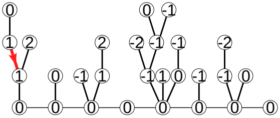
$(\mathfrak{f}, \mathfrak{l})$



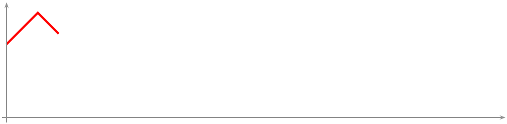
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



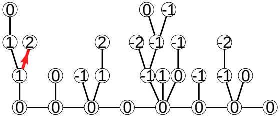
$(\mathfrak{f}, \mathfrak{l})$



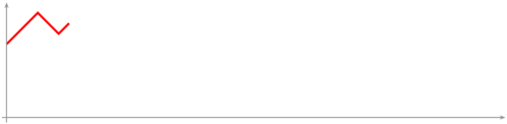
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



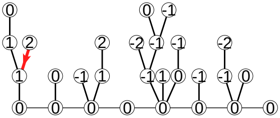
$(\mathfrak{f}, \mathfrak{l})$



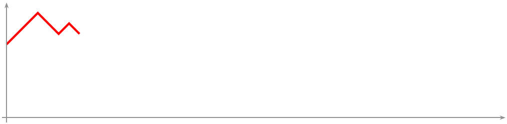
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



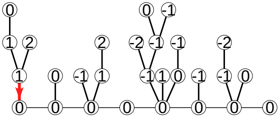
$(\mathfrak{f}, \mathfrak{l})$



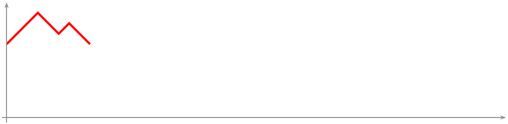
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



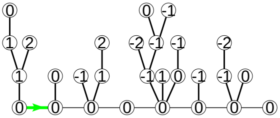
$(\mathfrak{f}, \mathfrak{l})$



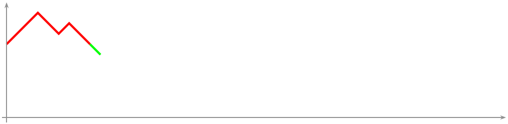
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



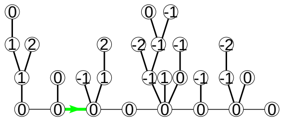
$(\mathfrak{f}, \mathfrak{l})$



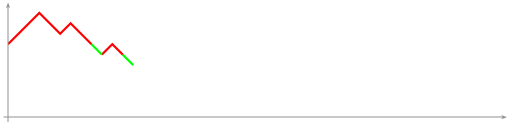
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



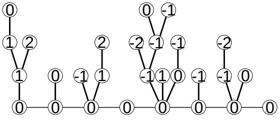
$(\mathfrak{f}, \mathfrak{l})$



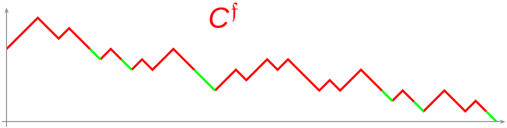
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function



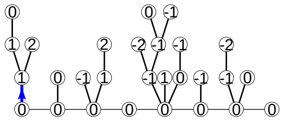
$(\mathfrak{f}, \mathfrak{l})$



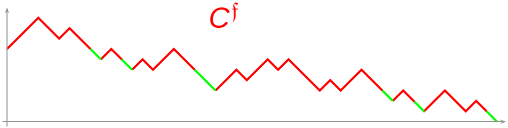
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function
- $L^{(\mathfrak{f}, \mathfrak{l})}$: labeling function



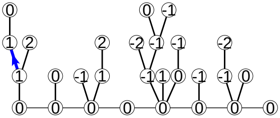
$(\mathfrak{f}, \mathfrak{l})$



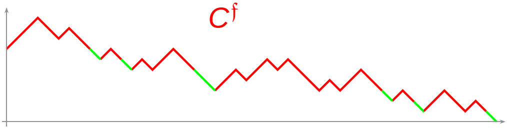
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function
- $L^{(\mathfrak{f},\mathfrak{l})}$: labeling function



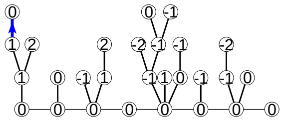
$(\mathfrak{f}, \mathfrak{l})$



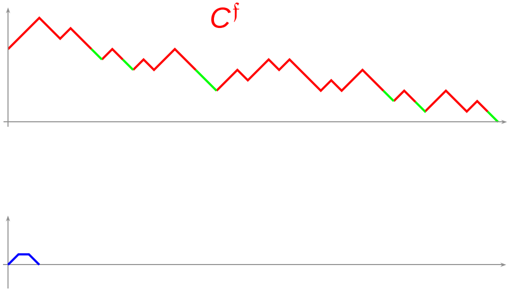
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function
- $L^{(\mathfrak{f},\mathfrak{l})}$: labeling function



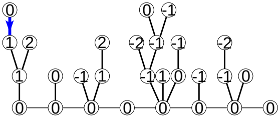
$(\mathfrak{f}, \mathfrak{l})$



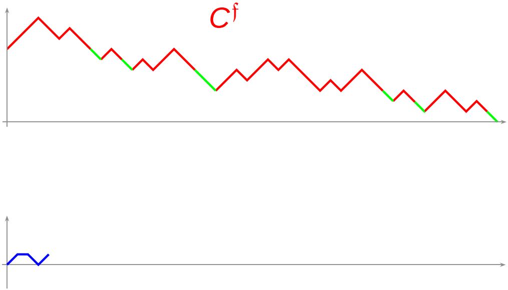
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function
- $L^{(\mathfrak{f},\mathfrak{l})}$: labeling function



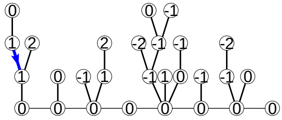
$(\mathfrak{f}, \mathfrak{l})$



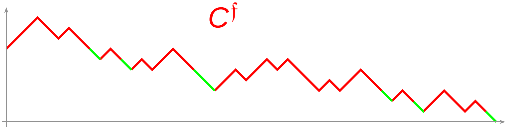
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function
- $L^{(\mathfrak{f},\mathfrak{l})}$: labeling function



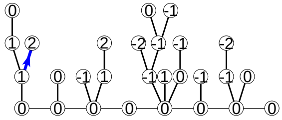
$(\mathfrak{f}, \mathfrak{l})$



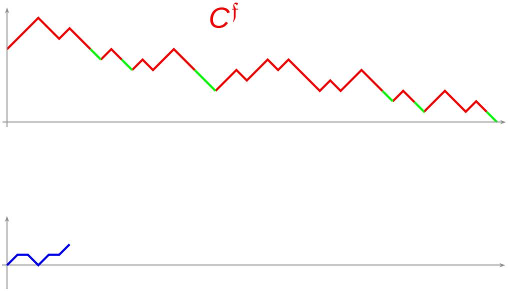
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function
- $L^{(\mathfrak{f},\mathfrak{l})}$: labeling function



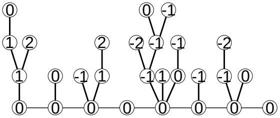
$(\mathfrak{f}, \mathfrak{l})$



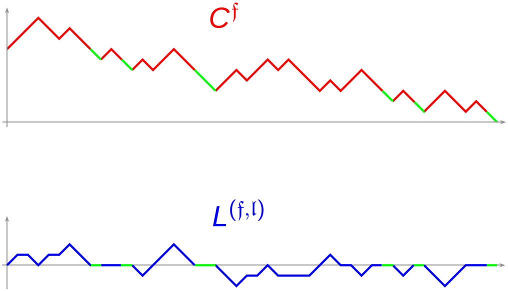
Contour pair

$(\mathfrak{f}, \mathfrak{l})$: well-labeled forest

- $C^{\mathfrak{f}}$: contour function
- $L^{(\mathfrak{f},\mathfrak{l})}$: labeling function



$(\mathfrak{f}, \mathfrak{l})$



Scaling limit of the contour pair

- $(\mathfrak{f}_n, \mathfrak{l}_n)$ uniform well-labeled forest with n edges and σ_n trees
- $(C_n, L_n) := \left(C^{\mathfrak{f}_n}, L^{(\mathfrak{f}_n, \mathfrak{l}_n)}\right)$ its contour pair

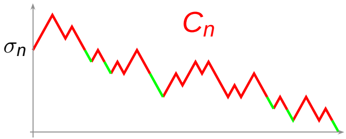
Proposition

Suppose that $\sigma_n/\sqrt{2n} \to \sigma \in]0, \infty[$. Then the process

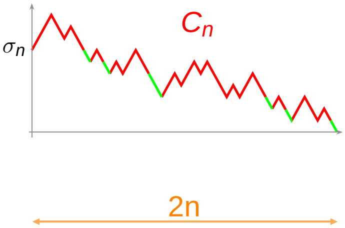
$$\left(\left(\frac{C_n(2ns)}{\sqrt{2n}}\right)_{0\leq s\leq 1}, \left(\frac{L_n(2ns)}{(8n/9)^{1/4}}\right)_{0\leq s\leq 1}\right)$$

converges weakly, for the uniform topology on $\mathscr{C}([0,1],\mathbb{R})^2$, toward the ***Brownian snake****'s head directed by a first-passage Brownian bridge.*

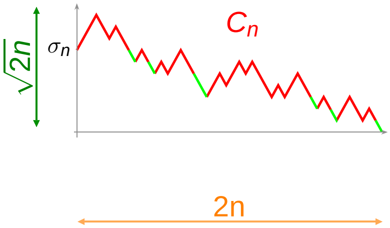
$$\left(\left(\frac{C_n(2ns)}{\sqrt{2n}} \right)_{0 \le s \le 1}, \qquad \right)$$



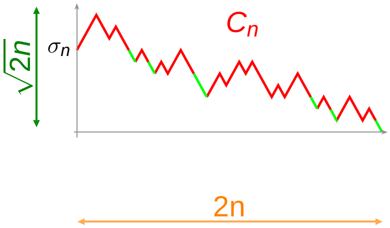
$$\left(\left(\frac{C_n(2ns)}{\sqrt{2n}} \right)_{0 \le s \le 1}, \qquad \right)$$

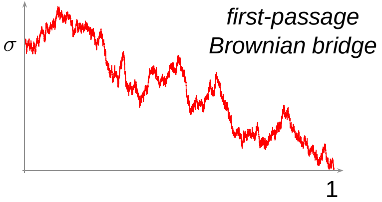


$$\left(\left(\frac{C_n(2ns)}{\sqrt{2n}} \right)_{0 \le s \le 1}, \quad \right)$$

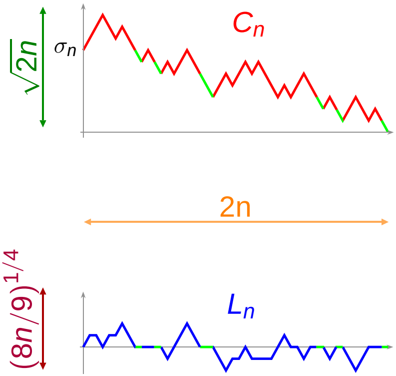


$$\left(\left(\frac{C_n(2ns)}{\sqrt{2n}} \right)_{0 \le s \le 1} , \qquad \qquad \right)$$

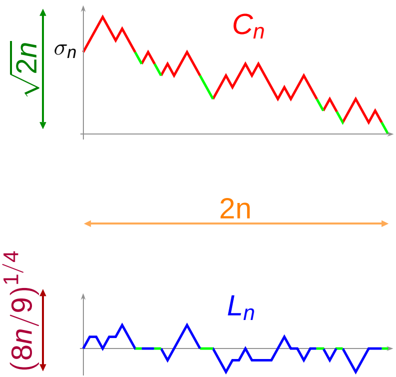


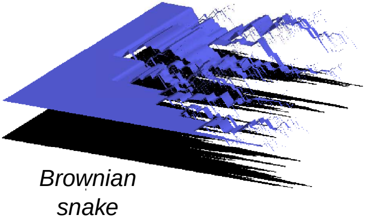


$$\left(\left(\frac{C_n(2ns)}{\sqrt{2n}}\right)_{0\leq s\leq 1}, \left(\frac{L_n(2ns)}{(8n/9)^{1/4}}\right)_{0\leq s\leq 1}\right)$$



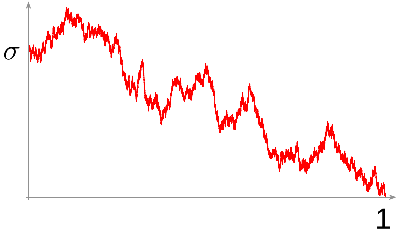
$$\left(\left(\frac{C_n(2ns)}{\sqrt{2n}}\right)_{0\le s\le 1},\ \left(\frac{L_n(2ns)}{(8n/9)^{1/4}}\right)_{0\le s\le 1}\right)$$





Brownian forests

The previous proposition allows to define a continuous version of well-labeled forest.

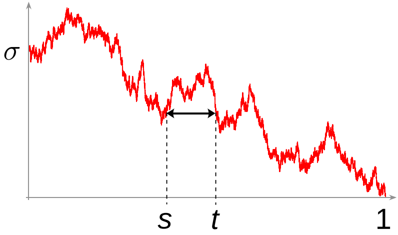


Quotient of $[0, 1]$:

- we identify the points *facing each other under the graph*

Brownian forests

The previous proposition allows to define a continuous version of well-labeled forest.

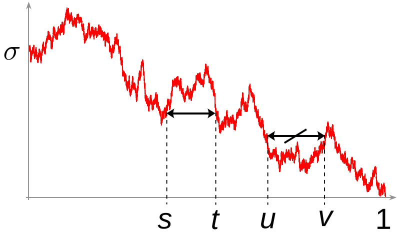


Quotient of $[0, 1]$:

- we identify the points *facing each other under the graph*
- $s \sim t$

Brownian forests

The previous proposition allows to define a continuous version of well-labeled forest.

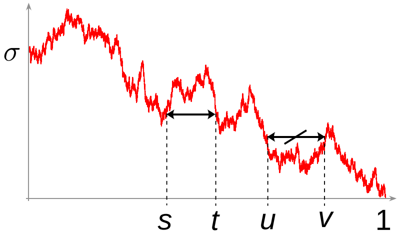


Quotient of $[0, 1]$:

- we identify the points *facing each other under the graph*
- $s \sim t$
- $u \not\sim v$

Brownian forests

The previous proposition allows to define a continuous version of well-labeled forest.

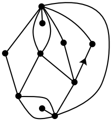


Quotient of $[0,1]$:

- we identify the points *facing each other under the graph*
- $s \sim t$
- $u \not\sim v$

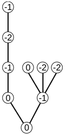
We endow this quotient with Brownian labels such that the variations along the "branches of the forest" are Brownian motions.

General principle

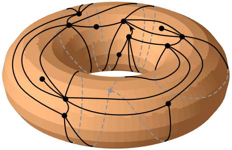


planar quad.

Cori–Vauquelin –Schaeffer ⟷

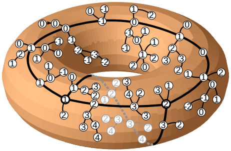


well-labeled tree

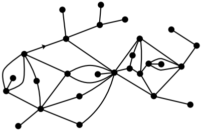


genus *g* bip. quad.

Chapuy–Marcus –Schaeffer ⟷

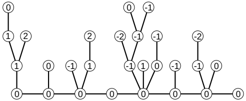


well-labeled *g*-tree



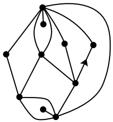
quad. with a bdry

Bouttier–Di Francesco –Guitter ⟷

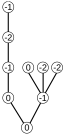


well-labeled forest
(+ bridge)

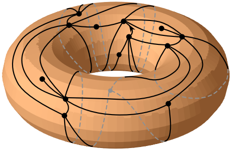
General principle



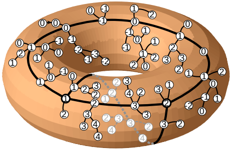
planar quad.



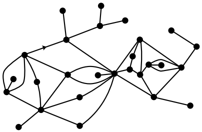
well-labeled tree



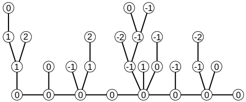
genus g bip. quad.



well-labeled g-tree

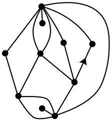


quad. with a bdry

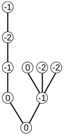


well-labeled forest
(+ bridge)

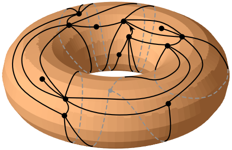
General principle



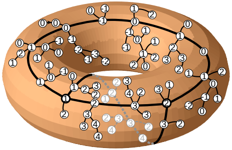
planar quad.



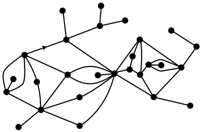
well-labeled tree



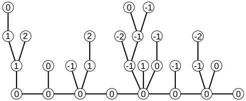
genus g bip. quad.



well-labeled g-tree

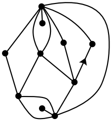


quad. with a bdry

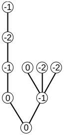


well-labeled forest
(+ bridge)

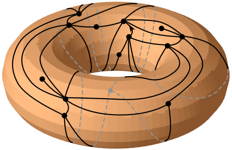
General principle



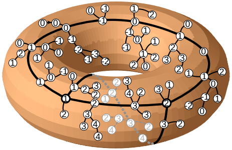
planar quad.



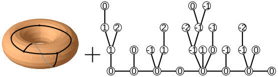
well-labeled tree



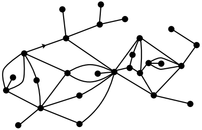
genus g bip. quad.



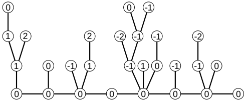
well-labeled g-tree



scheme +
well-labeled forests
(+ bridges)

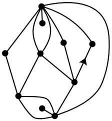


quad. with a bdry

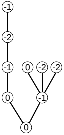


well-labeled forest
(+ bridge)

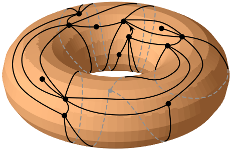
General principle



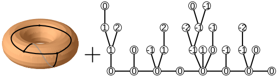
planar quad.



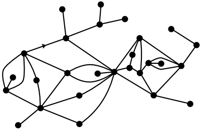
well-labeled tree



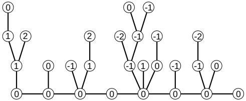
genus g bip. quad.



scheme +
well-labeled forests
(+ bridges)

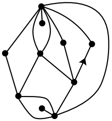


quad. with a bdry

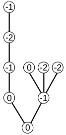


well-labeled forest
(+ bridge)

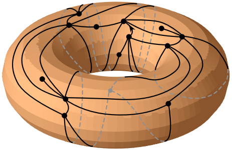
General principle



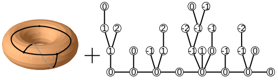
planar quad.



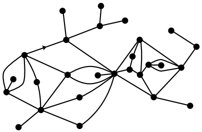
well-labeled tree



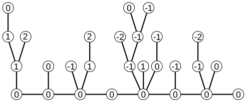
genus *g* bip. quad.



scheme +
well-labeled forests
(+ bridges)

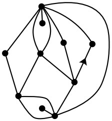


quad. with a bdry

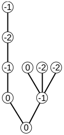


well-labeled forest
(+ bridge)

General principle

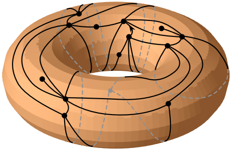


planar quad.

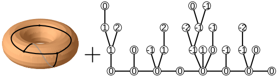


well-labeled tree

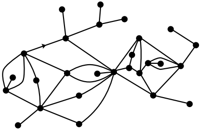
Continuum Random Tree



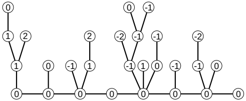
genus *g* bip. quad.



scheme +
well-labeled forests
(+ bridges)

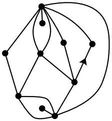


quad. with a bdry

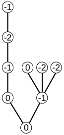


well-labeled forest
(+ bridge)

General principle

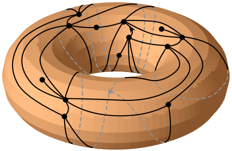


planar quad.

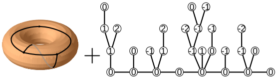


well-labeled tree

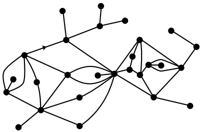
Continuum Random Tree
with Brownian labels



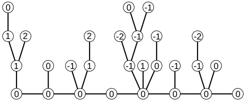
genus g bip. quad.



scheme +
well-labeled forests
(+ bridges)

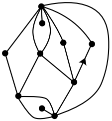


quad. with a bdry

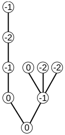


well-labeled forest
(+ bridge)

General principle

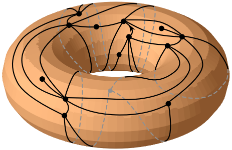


planar quad.

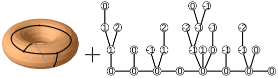


well-labeled tree

Continuum Random Tree with Brownian labels



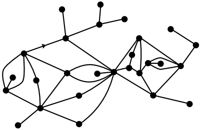
genus *g* bip. quad.



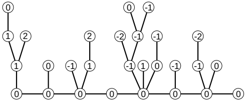
scheme + well-labeled forests (+ bridges)

Brownian g-tree

gluing of Brownian bridges then Brownian forest along the edges of a scheme

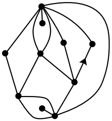


quad. with a bdry

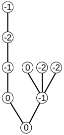


well-labeled forest (+ bridge)

General principle

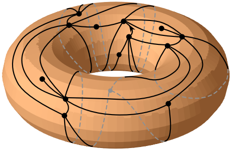


planar quad.

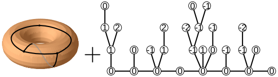


well-labeled tree

Continuum Random Tree
with Brownian labels



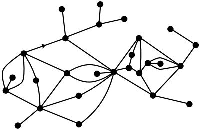
genus *g* bip. quad.



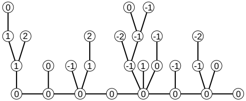
scheme +
well-labeled forests
(+ bridges)

Brownian g-tree

gluing of Brownian bridges
then Brownian forest along
the edges of a scheme



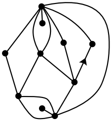
quad. with a bdry



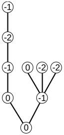
well-labeled forest
(+ bridge)

Brownian forest

General principle

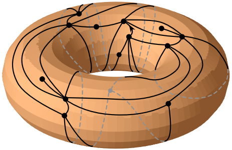


planar quad.

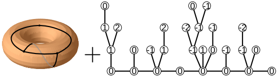


well-labeled tree

Continuum Random Tree
with Brownian labels



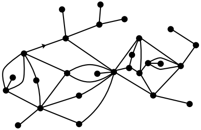
genus g bip. quad.



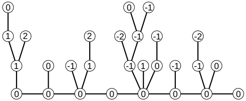
scheme +
well-labeled forests
(+ bridges)

Brownian g-tree

gluing of Brownian bridges
then Brownian forest along
the edges of a scheme



quad. with a bdry



well-labeled forest
(+ bridge)

Brownian forest
+
Brownian bridge
multiplied by $\sqrt{3}$

Thank you for your attention